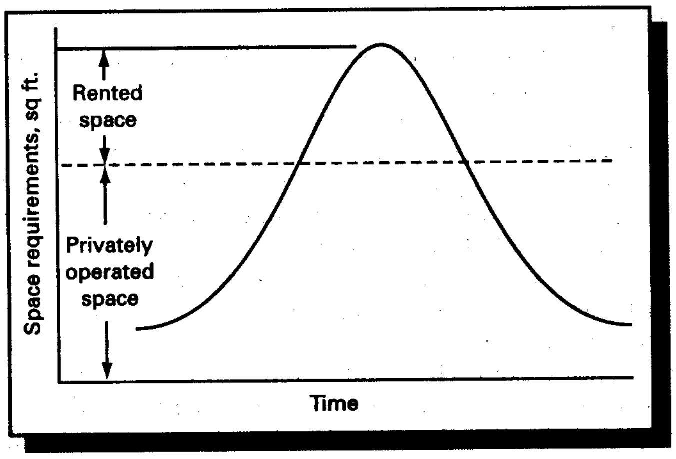
Figure 12-1
A Mixed Strategy of Rented and Privately Operated Warehouse Space for Variable Space Requirements



space requirements, there can be substantial space underutilization during a portion of the year. A better strategy would be to skim the space requirements so that a high level of utilization is realized and to use rented space on a short-term basis to meet peak space requirements. This strategy is graphically illustrated in Figure 12-1.

Finding the best mixed strategy is a matter of trying different sizes of privately operated space and determining the associated total cost for meeting all space needs throughout the year. Privately operated space is characterized by a combination of fixed and variable costs, whereas rented space is essentially an all-variable cost to the user. Thus, as privately operated space is increased in size, the combined cost will initially drop until the point where fixed costs and space underutilization of progressively larger warehouses causes total costs to increase. We seek the minimal cost point.

Example

Douglas-Biehl, a small chemical company, is planning to build a warehouse on the West Coast. Projections of average monthly demand on the warehouse are as follows:

Month	Demand, lb	Month	Demand, lb
Jan.	66,500	July	1,303,000
Feb.	328,000	Aug.	460,900
Mar.	1,048,500	Sept.	99,900
Apr.	2,141,000	Oct.	15,300
May	2,820,000	Nov.	302,200
June	2,395,000	Dec.	556,700
		Total	11,537,000

A *monthly* inventory turnover ratio[3] of 3, or 36 turns per year, is to be maintained for the warehouse. Of the total warehouse space, 50 percent is used for aisles, and only

[3]Monthly sales divided by average inventory.

70 percent is to be utilized to anticipate variability in space requirements. An average mix of chemical products occupies 0.5 cubic feet of space per lb and can be stacked 16 feet high on racks.

The warehouse, with equipment, can be constructed for $30 per square foot, amortized over 20 years, and operated at $0.05 per lb of throughput. Annual fixed costs are $3 per sq. ft. of total space. Space may be rented for a space charge of $0.10 per lb per month and an in-and-out handling charge of $0.07 per lb. What warehouse size should be constructed?

We first need to develop a space requirements table showing the space requirements throughout the year in square feet. We know from the turnover ratio that for every 3 lb of chemicals moving through the warehouse per month, 1 lb is maintained in inventory. For each pound stored, 0.5/16 sq. ft. of space is needed. Due to aisles, this space requirement needs to be doubled (1/0.50) and then increased for the rate of space utilization (1/0.70). Therefore, to convert demand to space requirements in square feet, we have

$$\begin{aligned}\text{Space (sq. ft.)} &= \text{Monthly demand (lb)} \times (1/3)(0.5/16)(1/0.50)(1/0.70)\\ &= \text{Monthly demand (lb)} \times 0.029762\end{aligned}$$

The space requirements table is developed in Table 12-2.

Next, we select a warehouse size to be tested. Let us try 60,000 sq. ft. A warehouse of this size costs $30/sq. ft. × 60,000 sq. ft. = $1,800,000 to build. Amortizing the construction cost over 20 years would yield an annual fixed cost of $90,000. A working cost table (Table 12-3) for this warehouse size alternative is developed. Repeating the same calculations for various warehouse sizes provides us with the data to develop the total annual cost curve as shown in Figure 12-2. The most economical warehouse size is 60,000 sq. ft. It can be anticipated that rented space will be needed during the months of April through June, with May being the peak month, requiring enough rented space to handle (2,820,000 × 0.29)/3 = 272,600 lb of chemicals.

Table 12-2 The Projected Space Requirements for Douglas-Biehl's West Coast Warehouse

Month	Warehouse Demand, lb	Space Requirements, sq. ft.	Month	Warehouse Demand, lb	Space Requirements, sq. ft.
Jan.	66,500	1,979[a]	July	1,303,000	38,780
Feb.	328,000	9,762	Aug.	460,900	13,717
Mar.	1,048,500	31,205	Sept.	99,900	2,973
Apr.	2,141,000	63,720	Oct.	15,300	455
May	2,820,000	83,929	Nov.	302,200	8,994
June	2,395,000	71,280	Dec.	556,700	16,568
			Totals	11,537,000	343,362

[a] 66,500 × 0.029762 = 1,979

Table 12-3 Costs for a Mixed Warehouse Size Strategy Using a 60,000 Square Foot, Privately Operated Warehouse

			Privately Operated			Rented			
Month	Warehouse Throughput, lb	Space Requirements, sq. ft.	Private Allocation	Monthly Fixed Cost	Monthly Variable Cost	Rented Allocation	Monthly Storage Cost	Monthly Handling Cost	Monthly Cost
Jan.	66,500	1,979	100%	$22,500[a]	$ 3,325[b]	0%	$ 0	$ 0	$ 25,825
Feb.	328,000	9,762	100	22,500	16,400	0	0	0	38,900
Mar.	1,048,500	31,205	100	22,500	52,425	0	0	0	74,925
Apr.	2,141,000	63,720	94[c]	22,500	100,627[d]	6	4,282[e]	8,992[f]	136,401
May	2,820,000	83,929	71	22,500	100,110	29	27,260	57,246	207,116
June	2,395,000	71,280	84	22,500	100,590	16	12,773	26,824	162,687
July	1,303,000	38,780	100	22,500	65,150	0	0	0	87,650
Aug.	460,900	13,717	100	22,500	23,045	0	0	0	45,545
Sept.	99,900	2,973	100	22,500	4,995	0	0	0	27,495
Oct.	15,300	455	100	22,500	765	0	0	0	23,265
Nov.	302,200	8,994	100	22,500	15,110	0	0	0	37,610
Dec.	556,700	16,568	100	22,500	27,835	0	0	0	50,335
Totals	11,537,000	343,362		$270,000	$510,377		$44,315	$93,062	$917,754

[a] [$90,000 + (3 × $60,000)]/12 = $22,500
[b] 66,500 × 0.05 = $3,325
[c] 60,000/63,720 = $0.94
[d] 2,141,000 × 0.94 × 0.05 = $100,627
[e] Given a monthly turnover ratio of 3 and 6% of the demand through the rented warehouse, then [(2,141,000 × 0.06)/3] × $0.10 = $4,282
[f] 2,141,000 × 0.06 × 0.07 = $8,992

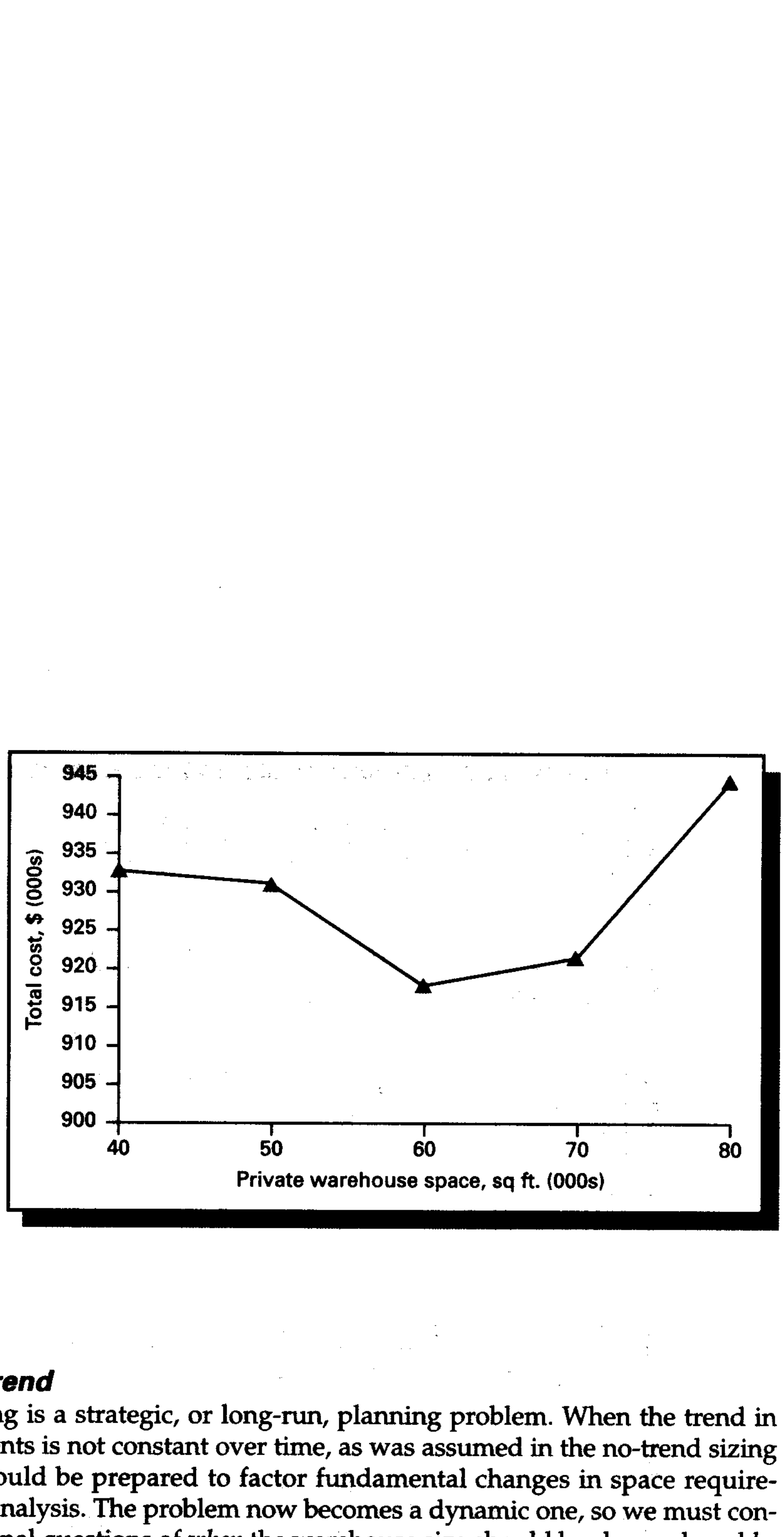


Figure 12-2 Total Annual Costs for Douglas-Biehl's Combined Strategy of Using Privately Operated and Rented Warehouse Space

Sizing with Trend

Warehouse sizing is a strategic, or long-run, planning problem. When the trend in space requirements is not constant over time, as was assumed in the no-trend sizing analysis, we should be prepared to factor fundamental changes in space requirements into our analysis. The problem now becomes a dynamic one, so we must consider the additional questions of *when* the warehouse size should be changed, and *by how much*. Determining the best warehouse size at any point in time requires balancing the benefits of being in a particular size with the costs of moving to another size. A methodology for this sizing problem is very similar to that presented for the dynamic location problem in Chapter 13. No further discussion of the methodology will be presented here.

Appraisal of the Sizing Method

The method for sizing a warehouse, although largely of a trial-and-error nature, offers some major benefits.

1. The method specifically draws attention to the problem of seeking the best privately operated warehouse size, in terms of a combination of owning and renting alternatives, rather than providing space in the form of either all privately operated space or all rented space.
2. The variability of space requirements due to seasonal fluctuations in supply and demand and the uncertainties associated with forecasting are considered.
3. The timing and magnitude of public warehouse space needs are defined and can be planned.
4. The timing and magnitude of private space needs are defined at present to permit lead time for planning and/or construction of space changes.

The method is not without its limitations. Chief among these are the following:

1. Inventory levels are used as the primary determinant of space needs. The space requirements of aisles, docks, staging areas, and order-picking areas are approximated and incorporated into the cost for a given warehouse size. They are not specifically treated. Therefore, the suggested size can only be an estimate of the final size to be built.

2. As with any dynamic model, long-range forecasts are required. Any errors in the plan, due to inaccurate forecasts, must be weighed against the alternative approach of changing the warehouse size as changes in space requirements are observed.

3. The selection of the sizing alternatives to be examined is based on judgment. As such, some size combinations may not be explored by the analysis. However, the improvement to be gained from additional size alternatives in the analysis should be minor.

Selecting the Space Type—Financial Considerations

Although seasonal fluctuation in space requirements plays a role in determining the space type to use, it is equally important to recognize that even when there is little seasonality in space needs, there is a choice of renting, leasing, or owning the space. The selection among these options is usually based on a financial comparison. Since the time horizon of the decision can be long, perhaps 20 years, the time value of money can be important in the selection process. That is, we want to compare the net present value of money according to

$$PV = \text{Lease payment} \times \frac{(1+i)^n - 1}{i(1+i)^n} \qquad \text{(12-1)}$$

where

NPV = net present value at time 0
I = initial investment, or cash outlay, at time 0
j = time period in the planning horizon between 0 and n
n = the time period at the end of the planning horizon
C_j = the cash flow difference (cash outflow) between alternatives in time period j
i = the discount rate, or hurdle rate, that such investments are expected to return annually
S_n = the cash return, or salvage value, of the asset at time period n

A positive *NPV* encourages investment, whereas a negative *NPV* discourages it. Alternately, *NPV* can be set to 0 and i found. This is the internal rate of return (*IRR*), which can then be compared to the company's hurdle rate. If *IRR* exceeds the hurdle rate, the investment is encouraged.

The present value formula [Equation (12-1)] is general and can be manipulated in many ways and applied to a broad range of financial problems. Only one example will be illustrated here.

Application[4]

A company's warehouse facilities have reached capacity in the mid-Atlantic region. Currently, the firm owns and operates two facilities in the area and utilizes approximately 150,000 sq. ft. of outside public warehouse space. The most pressing need is for overflow storage, which is now being handled by a public warehouse. Overflow requirements are expected to grow substantially in the next several years.

It is estimated that approximately 210,000 sq. ft. of space will be required. The alternatives have been reduced to the following: (1) Use public warehousing or (2) lease 210,000 sq. ft. for five years at $2.75 per sq. ft. per year with a five-year renewal option. Company federal taxes are at the rate of 39 percent per year.

For space of this size, the annual public warehousing charges are expected to be

Handling charges	$ 760,723
Storage charges	413,231
Total annual charges	$1,173,954

There are several categories of charges for the leased warehouse.

1 The estimated annual operating expenses are $309,914.

2. The annual lease payment is $577,500. According to one philosophy of financial analysts, the lease should be capitalized; that is, it should be treated as a debt or fixed asset. The company has an after-tax cost of capital of 10 percent. Discounting the ten equal lease payments to the present at a 10 percent discount rate gives $3,548,500. That is,

$$PV = \text{Lease payment} \times \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$= 577{,}500 \times \frac{(1+0.10)^{10} - 1}{0.10(1+0.10)^{10}}$$

$$= 577{,}500 \times 6.1446$$

$$= \$3{,}548{,}500$$

3. Other fixed assets and one-time charges for the leased facility.

Handling equipment	$170,800
Computer systems	26,740
Racks	252,000
Subtotal	$449,540
Startup costs	10,500
Total initial cash outlay	$460,040

4 Since all equipment is used up in ten years, there is no salvage value in it. The lease has no salvage value.

[4]Based on an example given in Thomas W. Speh and James A. Blomquist, *The Financial Evaluation of Warehousing Options: An Examination and Appraisal of Contemporary Practices* (Oxford, OH: The Warehousing Research Center, an affiliate of the Warehousing Education and Research Council, 1988), pp. 26–28.

The yearly cash flow difference between a public warehouse and a leased warehouse is \$1,173,954 − 309,914 = \$864,040, which we will call a *savings* (i.e., $-C_j$) to maintain the convention of Equation (12-1). Because of taxes, we need to account for depreciation on the assets. The depreciation schedule on the \$460,040 of initial outlay is

Year	Depreciation	Year	Depreciation
1	\$136,000	6	\$25,000
2	109,000	7	21,000
3	71,000	8	3,000
4	50,000	9	0
5	45,000	10	0

To determine the after-tax cash flow, consider the effect of taxes for the first year:

Savings	\$ 864,040
Depreciation	−136,000
Net profit (before tax)	\$ 728,040
Federal taxes (39%)	−283,936
Net profit (after tax)	\$ 444,104
Depreciation	+136,000
After-tax cash flow	\$ 580,104

Similar cash flow calculations can be made for each year (see Table 12-4).

We are now able to calculate the present value on the after-tax cash flow stream. Recalling that the after-tax hurdle rate is 10 percent and the discount formula is $1/(1 + i)^j$, we can calculate the following discounted cash flow stream:

Year	(1) After-Tax Net Cash Flow	(2) Discount Factor $1/(1 + 0.1)^j$	(3) = (1)(2) Discounted Net Cash Flow
0	(\$4009)		(\$4009)
1	580	0.9091	527
2	570	0.8264	471
3	555	0.7513	417
4	547	0.6830	374
5	545	0.6209	338
6	537	0.5645	303
7	535	0.5132	275
8	528	0.4665	246
9	527	0.4241	224
10	527	0.3855	203
		NPV =	(\$ 631)

The net present value is a *negative* \$631,000, meaning that the hurdle rate of 10 percent after taxes cannot be realized with a leased warehouse. Public warehousing should be used.

Table 12-4 Ten-Year Cash Flow Stream for Public Versus Leased Warehouse Comparison

Year	Savings: Lease vs. Public	Pre-Tax Net Cash Flow	Depreciation Schedule	Savings Less Depreciation	Taxes (39%)	Savings Less Depreciation & Tax	Savings Less Tax	After-Tax Net Cash Flow
0	$ 0	($4,009)[a]	$ 0					($4,009)
1	864	864	136	$ 728	$ 284	$ 444	$ 580[b]	580
2	864	864	109	755	294	461	570	570
3	864	864	71	793	309	484	555	555
4	864	864	50	814	317	497	547	547
5	864	864	45	819	319	500	545	545
6	864	864	25	839	327	512	537	537
7	864	864	21	843	329	514	535	535
8	864	864	3	861	336	525	528	528
9	864	864	0	864	337	527	527	527
10	864	864	0	864	337	527	527	527
Total	$8,640	$4,631	$460	$8,180	$3,189	$4,991	$5,451	$1,442

[a] Capitalized lease plus initial cash outlay, i.e., $3,548,500 + 460,040 = $4,008,540
[b] Add back depreciation, i.e., 444 + 136 = $580

Facility Configuration

Warehouses come in various shapes as well as various sizes. Any given warehouse size may be constructed in many different length, width, and height combinations. It is now assumed that the basic warehouse size has been established, and the next question is, What is the best configuration for the warehouse? A distinction is made between warehouses that are for general storage and handling and those that are used as crossdock, or high throughput, facilities.

Ceiling Height

In the previous sizing analysis, a given usable ceiling height was assumed. Determining this height for a medium throughput facility depends on construction costs, materials handling costs, and product load-stacking characteristics. If we were to double the ceiling height, thereby doubling the cubic content, the construction costs would not necessarily double. The roof and floor remain the same in both cases. Balancing construction costs, however, are the added materials handling costs due to the greater service time required for stacking and picking loads at a greater average height. Finally, the stacking characteristics of the stored goods can influence the desired ceiling height. Stability of the goods stacked individually in columns or in pallet-load units may put an upper limit on the height. Of course, using storage racks increases cube utilization and overcomes product-stacking limitations. Height limitations may then shift from the product characteristics to the characteristics of storage and materials handling equipment. Local building codes regarding sprinkler clearance may also influence the final ceiling height.

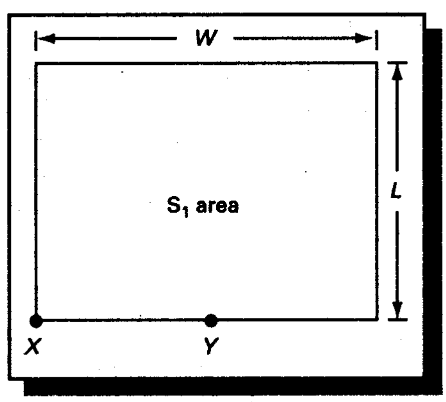
Choosing a ceiling height is a matter of trading off construction and equipment costs with materials handling costs in light of product, equipment, and legal constraints. In addition, there should be a minimum of extra space between the goods and the effective warehouse ceiling. The needed additional height is determined from an analysis of uncertain future requirements. In a general merchandise warehouse, product is typically stacked on racks about 16 feet high with the ceiling height at about 20 feet. There is no particular ceiling height limitation to storage warehouses or to those with automatic storage and retrieval systems. High throughput facilities such as cross docks or order-picking areas of distribution warehouses may limit stacking to one or two tiers with enough additional height to accommodate a fire protection system.

Length Versus Width

The length and width or configuration, of the warehouse building should be decided in relation to the materials handling costs of moving products through the warehouse and to the warehouse construction costs. Francis explored the question of configuration design in a theoretical way.[5] He examined configuration with the inbound-outbound dock located at X and then at Y, as shown in Figure 12-3. The warehouse uses rectangular aisles, stores n different item types, and has a floor area of S.

[5]Richard L. Francis, "On Some Problems of Rectangular Warehouse Design and Layout," *Journal of Industrial Engineering*, Vol. 18 (October 1967), pp. 595–604.

Figure 12-3
Outline of a Warehouse with Width *W* and Length *L* and with Possible Inbound-Outbound Dock Locations at *X* and *Y*



The optimum width W^* and length L^* are found by balancing materials handling costs against warehouse perimeter costs. Perimeter costs are defined as the annual construction and maintenance costs per foot of warehouse perimeter. For the dock located at X, Francis concluded, assuming out-and-back selection in a medium throughput facility, that the optimum width W^* is

$$W^* = \sqrt{\frac{C + 8k}{2C + 8k}}\sqrt{S} \qquad \textbf{(12-2)}$$

and the optimum length L^* is

$$L^* = \frac{S}{W^*} \qquad \textbf{(12-3)}$$

where

C = the sum of the total cost per foot to move an item of a given type in and out of storage multiplied by the expected number of items of a given type in and out of storage per year (dollar/ft.)
k = annual perimeter cost per foot (dollar/ft.)
S = required floor area of the warehouse (sq. ft.)

For the dock centered in the warehouse at location *Y*, the optimal width is

$$W^* = \sqrt{S} \qquad \textbf{(12-4)}$$

and the optimal length is

$$L^* = \sqrt{S} \qquad \textbf{(12-5)}$$

That is, the warehouse becomes square rather than rectangular. Of these two limiting cases, locating the dock in the center of the warehouse is the least expensive. Locating the dock at *X* has a total relevant cost TC_X of

$$TC_X = 2\sqrt{[(1/2)C + 2k][(1/4)C + 2k]}\sqrt{S} \qquad \textbf{(12-6)}$$

The relevant cost TC_Y for locating the dock at Y is

$$TC_Y = [(1/2)C + 4k]\sqrt{S} \quad \text{(12-7)}$$

The difference $TC_X - TC_Y$ is the premium that must be paid for locating the dock at X instead of at Y.

Example

A privately operated spare parts warehouse has a monthly throughput of 100,000 cases and an average in and out materials handling cost of $0.005 per foot per case moved. Order picking requires a trip to and from the outbound dock for each item requested. The total square footage needed for the operation is 300,000. Construction estimates show that a 500 × 600 sq. ft. warehouse can be built for $90 per sq. ft. The effective warehouse life is 20 years. The loading/unloading dock is to be located near to a corner of the proposed building. What are the best dimensions for the building and the total relevant cost?

The annual perimeter cost needs to be developed. There are 2(500) + 2(600) = 2,200 ft. in the perimeter. The construction cost is $90 × 300,000 = $27,000,000. Annualized, it is $27,000,000/20 = $1,350,000. On a perimeter-foot basis, it is $1,350,000/2,200 = $613.64/ft. This is k. C is 0.005 × 100,000 × 12 = $6000/ft.

To determine the warehouse width, Equation (12-2) applies. That is,

$$\begin{aligned} W^* &= \sqrt{\frac{6{,}000 + 8(613.64)}{2(6{,}000) + 8(613.64)}}\sqrt{300{,}000} \\ &= 440 \text{ ft.} \end{aligned}$$

and the length according to Equation (12-3):

$$L^* = 300{,}000/440 = 682 \text{ ft.}$$

The relevant cost for this rectangular warehouse from Equation (12-6) is

$$\begin{aligned} TC &= 2\sqrt{[(1/2)6{,}000 + 2(613.64)][(1/4)6{,}000 + 2(613.64)]}\sqrt{300{,}000} \\ &= 6{,}790.87(547.72) \\ &= \$3{,}719{,}495 \text{ per year} \end{aligned}$$

It should be noted that these formulas might not be valid where a conveyorized materials handling system is used, since conveyors decouple dock location and warehouse configuration from variable materials handling costs. Thus, conveyor systems can neutralize the disadvantages of multistory, L-shaped, or other configurations that deviate from the theoretical design.

Jenkins expanded the previous analysis by noting that when rail and truck docks are centered but at opposite ends of the building, the least expensive configuration is the square.[6] On the other hand, movement costs may not be the primary determinant of the warehouse dimensions.[7] Rather, the length of the warehouse may be dictated by the dock requirements for rail or truck. The long and narrow building configuration of LTL truck terminals is an example. The number of truck stalls for inbound and outbound production movement and the length of the siding necessary for efficient product flow would need to be compared with the theoretical findings. How to determine these dock dimensions is discussed in the section on dock design.

Configuring high throughput facilities, known as cross dock warehouses and transfer terminals, requires a different cost balance than for the typical warehouse. Cross docking is limited to receiving and shipping, eliminating storage and order picking activities of the typical warehouse. The function is to unload goods and immediately transfer them to another truck, which is as close to the receiving point as possible to minimize handling expense. Ideally, this would be to assign a shipping dock directly across from the receiving dock. This suggests that the best building design is a long, narrow rectangle, or I-shape, assuming no conveyors are used to move goods about the building.

Not all goods received at a particular dock are transferred to the dock immediately across from it due to the allocation of dock spaces and the breakdown of inbound merchandise destined for multiple destinations. From a materials handling standpoint, a building configuration has a *centrality index*, which is the weighted average distance that all goods move in the building. As the number of doors increases, the centrality index also increases. To reduce the index, and therefore the handling cost, alternatives to the I-shape can be used, such as the T-shape, L-shape, and H-shape. Although T-, L-, and H-shapes reduce centrality, their configuration has the disadvantage of losing some door spaces for trailers at the inside corners. Therefore, the basic trade-off that determines building configuration is a balance of freight handling cost and the cost of constructing a building with the needed number of doors. Research suggests that building shape depends on the number of doors needed.[8] The best shape for small to midsize cross docks is a rectangle, or I-shape. As building size increases to 150 to 250 doors, a T-shape is best. For buildings in excess of 250 doors, the H-shape is best.

Space Layout

Once certain decisions have been made concerning the general configuration of the warehouse, the next decision is to lay out the storage bays, shelves, and aisles. The problem is one of determining the number of slots to place along a shelf, the number of shelves to use, and whether the shelves should be placed parallel or perpendicular to the longest wall. Several formulas and decision rules have been developed to help make this decision.[9] Two of several configurations are discussed.

[6]Creed H. Jenkins, *Complete Guide to Modern Warehouse Management* (Upper Saddle River, NJ: Prentice-Hall, 1990), pp. 104–107.
[7]C. E. Hancock and H. F. Kraemer, "The Economic Sizing of Warehouses—A Comparison of Evaluation Models," a paper presented at the TIMS-ORSA Joint National Meeting, Minneapolis, October 7–9, 1964.
[8]John J. Bartholdi III and Kevin R. Gue, "The Best Shape for a Crossdock," Working paper.
[9]Joseph Bassan, Yaakov Roll, and Meir J. Rosenblatt, "Internal Layout Design of a Warehouse," *AIIE Transactions*, Vol. 12, No. 4 (December 1980), pp. 317–322.

Two possible shelving layouts are shown in Figure 12-4. The product is received through a door on one side of the building and is shipped out a door on the opposite side. An item requires four movements between a door and a storage location. Dock doors are located in the center of the building, and all parts of the warehouse have an equal likelihood of being utilized. Shelving is double-sided except for shelves against a wall, which are single-sided. The layout objective is to minimize the sum of materials handling cost, annual warehouse area cost, and annual cost associated with the size (perimeter) of the building. The following notation is useful:

w = width of double shelf (ft.)
L = length of storage space; for example, width of a pallet (ft.)
m = number of storage spaces along a shelf
h = number of storage levels in the vertical direction
n = number of double shelves; two single ones are considered as one double shelf
K = total warehouse capacity in storage spaces
a = width of an aisle (ft.), where all aisles are assumed to be of the same width
u = length of the warehouse (ft.)
v = width of the warehouse (ft.)
d = yearly throughput (demand) of the warehouse, in storage units (for example, pallets). It is assumed that a storage item occupies one space unit (items/yr.)
C_h = materials handling cost of moving a storage item one length unit (dollars/ft.)
C_s = annual cost per unit of warehouse area (heat, light, maintenance) (dollars/sq. ft.)
C_p = annual cost per unit length of external walls (dollars/ft.)

For layout 1, shown in Figure 12-4(a), the optimal number of shelf spaces should be

$$m_1^* = \frac{1}{L}\sqrt{\left[\frac{dC_h + 2aC_s + 2C_p}{2(dC_h + C_p)}\right]\left[\frac{K(w+a)L}{2h}\right]} \tag{12-8}$$

and the optimal number of double shelves is

$$n_1^* = \frac{1}{w+a}\sqrt{\left[\frac{2(dC_h + C_p)}{dC_h + 2aC_s + 2C_p}\right]\left[\frac{K(w+a)L}{2h}\right]} \tag{12-9}$$

The best warehouse configuration will have a length of

$$u_1 = n_1^*(w+a) \tag{12-10}$$

and a width of

$$v_1 = 2a + m_1^* L \tag{12-11}$$

For alternative layout 2, shown in Figure 12-4(b), the optimal parameters are

$$m_2^* = \frac{1}{L}\sqrt{\left[\frac{2dC_h + 3aC_s + 2C_p}{dC_h + 2C_p}\right]\left[\frac{K(w+a)L}{2h}\right]} \tag{12-12}$$

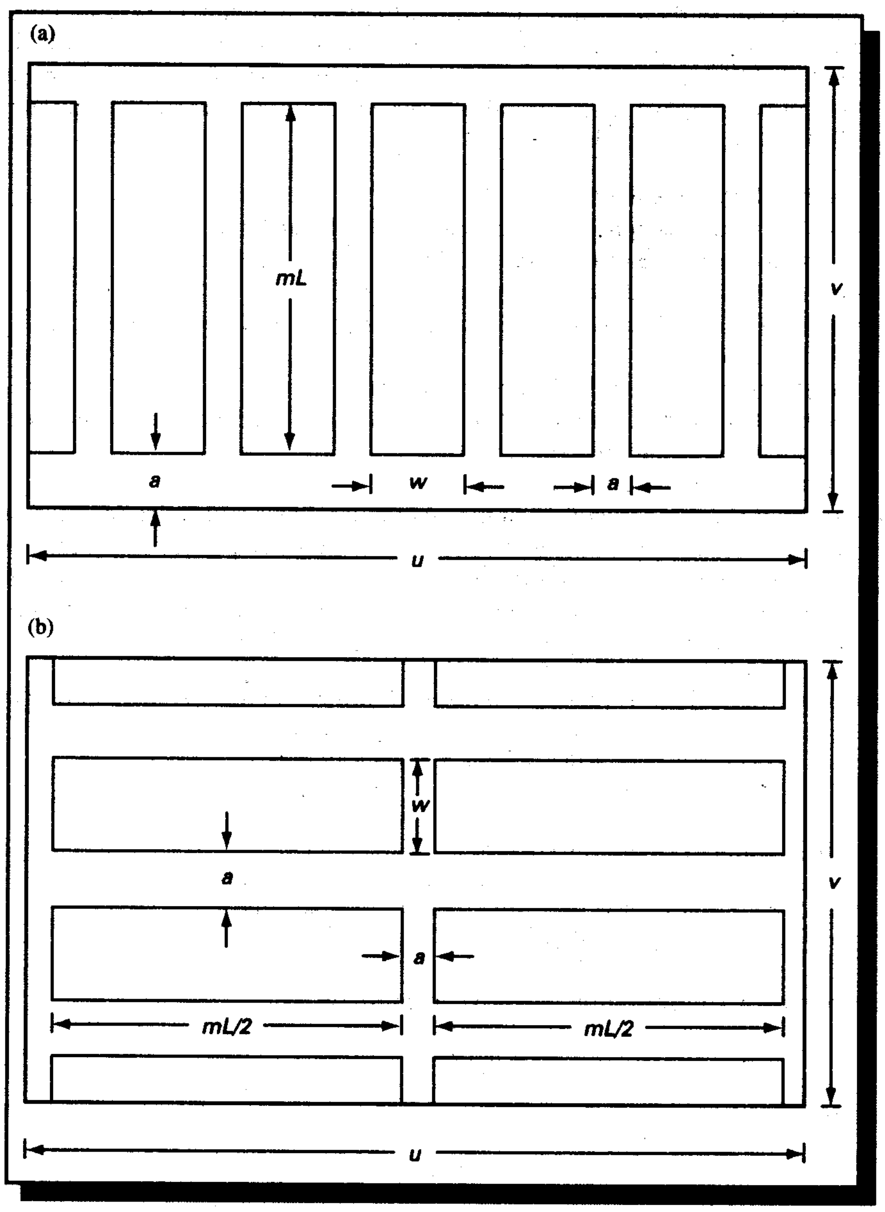


Figure 12-4 Aerial View of Two Possible Layouts of Shelving in a Rectangular Warehouse Configuration

and

$$n_2^* = \frac{1}{w+a}\sqrt{\left[\frac{dC_h + 2C_p}{2dC_h + 3aC_s + 2C_p}\right]\left[\frac{K(w+a)L}{2h}\right]} \quad (12\text{-}13)$$

where

$$u_2 = 3a + m_2^* L \quad (12\text{-}14)$$

and

$$v_2 = n_2^* (w+a) \quad (12\text{-}15)$$

To minimize costs between these two layout choices, the following decision rule can be applied: If $d < C_p/C_h$, then layout 1 is preferred to layout 2. If $d > 2C_p/C_h$, then layout 2 is preferred. However, if $C_p/C_h < d < 2C_p/C_h$, no conclusion can be drawn.

Example

Suppose that a warehouse is to be configured according to the layout in Figure 12-4(b). The building is to handle a throughput of 400,000 pallets per year. These pallets require storage space of 4 × 4 × 4 ft., and can be stacked four pallets high. Back-to-back pallet racks are 8 ft. wide. Aisles are 10 ft. wide. The materials handling cost is $0.001 per ft., annual space costs are $0.05 per sq. ft., and the annual cost per foot of perimeter wall is $3. The warehouse turnover is eight times per year with total warehouse capacity of 50,000 slots. What size building should be planned?

First, the number of storage spaces for the longitudinal bay, according to Equation (12-12), should be

$$m^* = \frac{1}{4}\sqrt{\left[\frac{2(400{,}000)(0.001) + 3(10)(0.05) + 2(3.00)}{400{,}000(0.001) + 2(3.00)}\right]\left[\frac{50{,}000(8+10)(4)}{2(4)}\right]}$$
$$= 237 \text{ spaces}$$

The number of double storage racks, according to Equation (12-13), should be

$$n^* = \frac{1}{8+10}\sqrt{\left[\frac{400{,}000(0.001) + 2(3.00)}{2(400{,}000)(0.001) + 3(10)(0.05) + 2(3.00)}\right]\left[\frac{50{,}000(8+10)(4)}{2(4)}\right]}$$
$$= 26 \text{ racks along one side of the warehouse.}$$

The length and width of the warehouse from Equation (12-14) and Equation (12-15) should be:

$$u = 3(10) + 237(4) = 978 \text{ ft. of length}$$

and

$$v = 26(8+10) = 468 \text{ ft. of width}$$

Dock Design

Dock design begins with the need for a rail or a truck dock at the warehouse. Nearly every warehouse requires at least one truck dock. The need for a rail dock is not as universal and depends on whether the product is to be received or shipped in large enough quantities to justify rail movement. Even if there is a need, a rail siding may not be possible if the warehouse is not located on a rail spur and the railroad is not willing to provide a spur. For purposes of discussion, let us assume that both dock types are needed.

Rail Dock

A primary consideration in dock design is the dock length that is needed to handle the product flow efficiently. Preliminary estimates can be made by multiplying the total average demand by the average length of rail cars used and dividing this quantity by the average quantity stored in the average rail car multiplied by the number of car changes per day. That is,

$$L = \frac{DS}{QN} \qquad \textbf{(12-16)}$$

where:

L = length of rail dock needed (ft.)
D = daily demand from all orders (cwt./day)
S = length of the average rail car used (ft.)
Q = average product weight placed in each car (cwt./car)
N = number of car changes per day (number/day)

Example

A food warehouse receives by rail, on the average, 14,000 cwt. of merchandise per day. The railcars have a capacity of 570 cwt. for this type of merchandise and their effective length is 75 ft. Two car changes along the siding can be completed each day.

The length of rail siding needed can be estimated from Equation (12-16) as

$$L = \frac{14{,}000(75)}{570(2)} = 921 \text{ ft.}$$

In addition to dock length, there are several other dock design considerations. For example, should the extra expense of enclosing the dock be incurred? An enclosed dock provides weather protection, gives some protection against theft, and contributes to labor efficiency in loading and unloading. The required platform depth is another question. If forklift trucks are to be used for loading and unloading, a minimum of 12 feet is necessary for safe maneuvering. If the dock is also to serve as a staging area for temporary holding while checking the incoming order or palletizing the received goods, a much greater dock depth is required, perhaps 40 or 50 feet. Finally, the level of the dock in relation to the rail car bed is of concern. Either the dock level must be raised to meet the car bed, or the car bed must be lowered to the dock level. Because only slight inclines are possible with most materials handling equipment and because of the expense of raising the entire warehouse floor, it is usually more economical to recess the tracks below the dock level. A well of 45 inches puts the car bed at the dock level. The gap between the car bed and the dock is bridged using a steel dock plate.

Truck Dock

Most of the factors affecting truck dock design are the same as those affecting rail dock design. However, instead of computing a length of dock, truck docks are frequently referred to as the number of dock doors, or stalls, required. Of course, a truck door has a standard width that can be converted to the total required dock length. Very simply, the number of truck dock doors needed can be found by

$$N = \frac{DH}{CS} \qquad (12\text{-}17)$$

where

N = total number of truck dock doors
D = average dock throughput per day
H = time required to load or unload a truck
C = truck capacity
S = time per day available to load or unload trucks

This formula calculates the average number of truck doors. It does not account for the variation in trucks available for loading and unloading, dock throughput, or the truck loading and unloading rate. Some additional doors may be needed to meet these uncertainties.

Example

A warehouse for Rico Discount Drug Stores replenishes 250 drug retail stores in its region on a weekly basis. The average store order is 6,500 lb and 4 stores' orders can be placed on a single truck. It takes two workers a total of two hours to load a truck and they work an eight-hour shift. Rico assigns as many workers as may be needed to load the trucks in eight hours. How many truck doors are needed to meet this average level of activity?

We can estimate that 50 stores are served each day of a five-day workweek. Therefore, 50 × 6,500 = 325,000 lb of merchandise is picked and loaded for delivery to stores each day. If orders for four stores are placed on a truck, a truckload is 4 × 6,500 = 26,000 lb. Using Equation (12-17), we can estimate the number of doors as

$$N = \frac{325{,}000(2)}{26{,}000(8)} = 3.15, \text{ or 4 doors}$$

Four doors allow Rico extra capacity for contingencies. In fact, with the throughput of 26,000(8)(4) = 416,000 for 4 doors in total, there is a (416,000 − 325,000) ×

truck and pallet system, a conveyorized system, an automated storage and retrieval system, or some combination of these system. Choosing among these can begin with a financial analysis similar to that used for selecting a warehouse type. The final choice must be tempered with subjective considerations such as risk, flexibility, and obsolescence.

Example

A manufacturer of office copying equipment is to construct a warehouse for spare parts. The choices for the internal materials handling system design reduce to a forklift truck and pallet system with conveyorized order picking and an automated storage and retrieval system. The company projects 3,000,000 orders to be picked per year and expects a return on projects of 20 percent per year before taxes.

The conveyor and forklift truck system requires an investment in racks of \$2,000,000 and trucks and conveyors of \$1,500,000. Racks have a 20-year life and a salvage value of 30 percent of their initial value at the end of 20 years. The trucks and conveyors have a 10-year life with a 10 percent salvage value at the end of 10 years. The throughput cost is \$0.50 per order.

The automatic storage and retrieval system requires an investment in racks of \$3,000,000 and \$2,000,000 in equipment and controls. The racks have a 20-year life with a 30 percent salvage value at the end of that life. The equipment and controls have a 10-year life with a 10 percent salvage value at the end of 10 years of use. The throughput cost is \$0.10 per order.

A financial analysis is conducted to determine the preferred alternative. We wish to compare the net present value of each alternative according to Equation (12-1). However, Equation (12-1) is modified slightly to account for the different life spans of racks versus equipment, and the term C_j represents a cost (cash outflow) and not a savings (cash inflow). That is, the NPV for racks is

$$NPV = -I - \frac{C_j}{(1+i)^j} + \frac{S_{20}}{(1+i)^{20}}$$

whereas for the equipment, *NPV* is

$$NPV = -I - \frac{C_j}{(1+i)^j} + \frac{S_{10}}{(1+i)^{10}} - \frac{I}{(1+i)^{10}} - \frac{C_{j+10}}{(1+i)^{j+10}} + \frac{S_{20}}{(1+i)^{20}}$$

We judge the best alternative to be the one having the least negative *NPV*.

Two tables are now developed—one for the conveyor and forklift truck system (Table 12-5), and one for the AS/RS (Table 12-6). Since the *NPV* for the AS/RS is less negative than that for the conveyor and forklift truck system, the AS/RS offers the better return (costs less) and is the one that should be considered for implementation.

Year	Investment: Racks	Investment: Equipment	Annual Operating Cost	Cash Flow	Discounted Cash Flow[d]
0	($2,000)	($1,500)		($ 3,500)	($ 3,500)
1			($1,500)	(1,500)	(1,250)
2			(1,500)	(1,500)	(1,042)
3			(1,500)	(1,500)	(868)
4			(1,500)	(1,500)	(723)
5			(1,500)	(1,500)	(603)
6			(1,500)	(1,500)	(502)
7			(1,500)	(1,500)	(419)
8			(1,500)	(1,500)	(349)
9			(1,500)	(1,500)	(291)
10		(1,350)[a]	(1,500)	(2,850)	(460)
11			(1,500)	(1,500)	(202)
12			(1,500)	(1,500)	(168)
13			(1,500)	(1,500)	(140)
14			(1,500)	(1,500)	(117)
15			(1,500)	(1,500)	(97)
16			(1,500)	(1,500)	(81)
17			(1,500)	(1,500)	(68)
18			(1,500)	(1,500)	(56)
19			(1,500)	(1,500)	(47)
20	600[b]	150[c]	(1,500)	(750)	(20)
				NPV =	($11,003)

[a]Equipment is replaced with a net investment equal to the cost of new equipment less the salvage value of the old; i.e., $1,500,000 - (1,500,000)(0.10) = $1,350,000.

[b]Salvage value of $2,000,000(0.30) = $600,000.

[c]Salvage value of $1,500,000(0.10) = $150,000.

[d]Cash stream discounted at 20% according to $1/(1 + 0.2)^j$.

Table 12-5 Cash Flow Analysis for the Conveyor and Forklift Truck Materials Handling Alternative

On a smaller scale than the entire materials handling system, individual pieces of equipment vary in their capacities and capabilities. Each has a different initial investment, annual operating expense, and salvage value. Again, selection is by comparing present values of the alternatives. When operating expenses are equal for all years over the useful life, and the useful life of the equipment is the same among alternatives, the net present value equation can be rewritten as follows:

$$NPV = I + C\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{S_n}{(1+i)^n} \tag{12-18}$$

	Investment Annual				
Year	Racks	Equipment	Operating Cost	Cash Flow	Discounted Cash Flow[d]
0	($3,000)	($2,000)		($5,000)	($5,000)
1			($ 300)	(300)	(250)
2			(300)	(300)	(208)
3			(300)	(300)	(174)
4			(300)	(300)	(145)
5			(300)	(300)	(121)
6			(300)	(300)	(100)
7			(300)	(300)	(84)
8			(300)	(300)	(70)
9			(300)	(300)	(58)
10		(1,800)[a]	(300)	(2,100)	(339)
11			(300)	(300)	(40)
12			(300)	(300)	(34)
13			(300)	(300)	(28)
14			(300)	(300)	(23)
15			(300)	(300)	(19)
16			(300)	(300)	(16)
17			(300)	(300)	(14)
18			(300)	(300)	(11)
19			(300)	(300)	(9)
20	900[b]	200[c]	(300)	800	21
				NPV =	($6,722)

[a]Equipment is replaced with a net investment equal to the cost of new equipment less the salvage value of the old; i.e., $2,000,000 - (2,000,000)(0.10) = $1,800,000.
[b]Salvage value of $3,000,000(0.30) = $900,000.
[c]Salvage value of $2,000,000(0.10) = $200,000.
[d]Cash stream discounted at 20% according to $1/(1 + 0.2)^i$.

Table 12-6 Cash Flow Analysis for the AS/RS Materials Handling Alternative

where

NPV = net present value of equipment over its useful life
I = initial investment
C = annual operating cost
i = the discount, or hurdle, rate that such investments are expected to return
S_n = salvage value in year n
n = useful life of the equipment (years)

For convenience, the sign convention has been reversed from the previous example. The goal now is to select the alternative with the minimum net present value.

Example

Suppose that two type *A* forklift trucks can move the same amount of goods as three type *B* trucks. The following additional data are available:

	Two Type A Trucks	Three Type B Trucks
Total initial investment	$20,000	$15,000
Useful life (planned)	7	7
Salvage value (estimated)	$ 5,000	$ 2,000
Annual operating expenses	$ 4,000	$ 6,000
Hurdle rate	0.20	0.20

Applying Equation (12-18) to both truck types, we have

$$NPV_A = 20{,}000 + 4{,}000\frac{(1+0.2)^7 - 1}{0.2(1+0.2)^7} - \frac{5{,}000}{(1+0.2)^7} = \$33{,}023 \longleftarrow \text{Best choice}$$

and

$$NPV_B = 15{,}000 + 6{,}000\frac{(1+0.2)^7 - 1}{0.2(1+0.2)^7} - \frac{2{,}000}{(1+0.2)^7} = \$36{,}040$$

Since $NPV_A < NPV_B$, selecting two trucks of type *A* seems to be the best financial choice.

Equipment Replacement

Materials handling equipment frequently has a shorter life than storage racks, bins, mezzanines, and other nonmechanical devices used in the handling process. Therefore, it is often necessary to develop a policy to replace equipment when it wears out or becomes obsolete. The need for a replacement policy is quite clear in the case of forklift trucks, where the economic life is not long and they must be replaced often. The need for a policy also occurs in various segments of bulk-handling systems or conveyor systems, where the useful life of the equipment may be much longer. It is common for management to have replacement rules of thumb, such as to replace forklift trucks every five years. Rules of thumb based on experience may be quite good. However, when such experience is not available to help develop policy guidelines, or when these rules of thumb have not been tested by "hard" economic analysis, it is useful to have an analytical means of developing replacement policies.

For developing replacement policies, special forms of present value analysis can be useful, although other methods such as payback and simple return on investment may be employed. There are several key features to note about such problems. First, the replacement cycle is expected to continue indefinitely into the future. Second, equipment-operating costs tend to increase over the years as equipment ages. Third, subsequent equipment is more efficient as technological improvements occur. To compare a stream of replacement cycles of different lengths, a form of present value analysis known as *equivalent annual cost* (*AC*) is used. That is,

$$AC_n = \left[I + \sum_{j=1}^{n} \frac{C_j}{(1+i)^j} - \frac{S_n}{(1+i)^n} \right] \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \qquad (12\text{-}19)$$

The period n for replacement that gives the minimum AC_n value is sought.

Example

Suppose a fleet of specialized materials handling trucks is used in a warehouse. Trucks are continually being replaced at an initial cost of $30,000 each. The salvage value declines proportionately with the age of a truck such that $S_n = I(1 - R \times n)$, where R is $1/N$, N is the normal life of a truck, and n is the replacement cycle time. N is 10 years for these trucks. Trucks can be sold at any time for the net undepreciated value. Operating cost for a truck, including maintenance, is $2,000 during the first year and tends to increase at the rate of $300 per year squared after the first year. However, because of technological improvement, it is expected that there will be a $200 per year reduction in operating expenses. A 20 percent return before taxes is the guideline on all company projects.

The operating cost for a truck, including the effect of technological improvements, can be approximated as $C_j = a + b(j-1) + c(j-1)^2$ where a = constant level of annual operating costs (dollars), b = rate of increase (or decrease) in annual operating costs due to technological improvements (dollars/year), c = rate of increase in annual operating costs (dollars/year/year), and j = the particular year of the cost estimate. Using this C_j cost function, as well as other data about the problem, we can compute the equivalent annual cost for a one-year ($n = 1$) replacement cycle. That is,

$$AC_1 = \left[30{,}000 + \sum_{j=1}^{1} \frac{2{,}000 - 200(0) + 300(0)^2}{(1+0.2)^1} - \frac{27{,}000}{(1+0.2)^1} \right] \left[\frac{0.2(1+0.2)^1}{(1+0.2)^1 - 1} \right] = \$11{,}000$$

Repeating this type of calculation for increasing values of n produces the series of annual cost values shown in Table 12-7. The lowest equivalent annual cost is for $n = 3$. Thus, this suggests that to minimize costs, the best policy is to replace the fork-lift trucks at the end of three years of service, but replacing trucks between two and five years of service results only in costs that are a maximum of 3 percent greater than optimum.

Product Layout Decisions

An important warehouse design decision concerns the internal layout of the items. After a building configuration is known; after receiving and shipping facilities are

Table 12-7 Example of Calculations for Determining Optimum Equipment Cycle Time[a]

Replacement Cycle Time, n	(1) Initial Investment, I	(2) Total Operating Costs, C_j	(3) Discounted Operating Costs, $\sum_{j=1}^{n} \frac{C_j}{(1+i)^j}$	(4) Salvage Value, S_n	(5) Discounted Salvage Value, $\frac{S_n}{(1+i)^n}$	(6) Discounted Factor, $\frac{i(1+i)^n}{(1+i)^n - 1}$	(7) = (1+3–5)(6) Equivalent Average Annual Cost, AC_n
1	$30,000	$2,000[b]	$1,667	$27,000[c]	$22,500	1.20	$11,000
2	30,000	4,100	3,125	24,000	16,667	0.65	10,698
3	30,000	6,900	4,745	21,000	12,153	0.47	**10,618** ←
4	30,000	11,000	6,722	18,000	8,680	0.39	10,936
5	30,000	17,000	9,133	15,000	6,028	0.33	10,925
6	30,000	25,500	11,979	12,000	4,019	0.30	11,388
7	30,000	37,100	15,216	9,000	2,512	0.28	11,957
8	30,000	52,400	18,774	6,000	1,395	0.26	12,319
9	30,000	72,000	22,572	3,000	581	0.25	12,998
10	30,000	96,500	26,528	0	0	0.24	13,567

[a]All costs in thousands of dollars
[b]Computed as $C_j = 2000 - 200(j-1) + 300(j-1)^2$ and accumulated when there is more than one year in the replacement cycle
[c]Computed as $S_n = I[1 - 0.1(n)]$

specified; after space blocks are defined for hazardous products, for products under theft protection, and for order picking; and after considering the materials handling system to be used, decisions need to be made as to where stock items are to be located, how they should be arranged, and what method should be used for finding stock in the warehouse. These questions have long concerned the industrial engineer in the layout of production facilities, and much of the decision methodology developed for production layout is transferable to the warehouse layout problem. Such methods supplement those dealing more directly with the layout problem in the warehouse, and these methods are blended into the following discussion.

Stock Location

Stock location is the problem of deciding the physical layout of merchandise in a warehouse to minimize materials handling expenses, to achieve maximum utilization of warehouse space, and to meet certain constraints on merchandise location such as for security, fire safety, product compatibility, and order-picking needs. Stock retrieval (or placement) generally occurs in three ways. First, there is out-and-back selection, where only one item or load is picked from a particular location. A typical trip would be to go out from the outbound dock, pick a product, and return to the outbound dock.

Second, there is picker routing, where several items on an order are picked before returning to the outbound point, or staging area. The volume picked on any one route may be limited by the truck capacity of the order picker.

Third, there is a designated order-picking area per worker. Order pickers retrieve items by out-and-back selection or picker routing within the limits of their specified work areas.

The objective of location planning in each of these problems is to minimize the total handling costs. This often translates into minimizing the total travel distance throughout the warehouse. In addition, order picking is typically of greater concern than item storage because the labor expense to pick merchandise from a warehouse is much greater than that required to store it. This is due to the smaller average load sizes moving from a storage location than moving to it. Therefore, our primary concern is with minimizing materials handling costs in the order picking activity of a warehouse.

Intuitive methods have appeal in that they provide some useful guidelines for layout without the need for higher-level mathematics. Layout is often intuitively based on four criteria: complementarity, compatibility, popularity, and size. *Complementarity* refers to the idea that items often ordered together should be located near each other. Examples of such items are paint and brushes, razor blades and shaving cream, and pens and pencils. This factor is particularly important when order picking is of the picker-routing type or when laying out storage, or flow, racks in designated order-picking area systems.

Compatibility includes the question as to whether items can be practically located next to each other. Auto tires are not compatible with foodstuffs, and gasoline is not compatible with cylinders of oxygen. Therefore, they should not be located near each other. Products are considered compatible if there is no restriction on their location proximity.

Compatibility and complementarity may be decided before order-picking costs are taken into consideration. In addition, there is concern with balancing workloads, minimizing fatigue, and equalizing travel distance when multiple workers are used to fill orders, as in a designated order-picking area design. Once these restrictions have been taken into account, layout by popularity or by size becomes appropriate.

Layout by *popularity* recognizes that products have different turnover rates in a warehouse, and materials handling cost is related to the distance traveled in the warehouse to locate and pick the stock. If stock is retrieved from a location in smaller volumes per trip than it is supplied, materials handling costs can be minimized by locating the fast-moving items close to the outbound point, or staging area, and the slower-moving items to the rear of these. This assumes that the items requiring a large number of trips for a given level of demand will have the shortest possible travel distance per order-picking trip.

Layout by popularity neglects the size of the item being stored and the possibility that a larger number of smaller items can be located near the outbound point, or staging area. This suggests that handling costs might be minimized if the *size* (cubic volume) of the item is used as the layout guide. By locating the smaller items near the outbound point in the warehouse, materials handling may be less than in the arrangement by popularity, as a greater density of items can be located close to the shipping dock.

However, layout by size does not guarantee lower costs than layout by popularity. The by-size method would be a good choice when high turnover is concentrated in the smaller items.

Layout by popularity or by size is not completely satisfactory because one neglects an important factor of the other. Heskett combined both features into a cube-per-order index.[13] *The index is the ratio of the average required cubic footage of the product for storage to the average number of daily orders on which the item is requested.* Products having *low* index values are located as near as possible to the outbound point. The cube-per-order index (COI) attempts to load the warehouse space so that the greatest volume of stock moves the shortest possible distance. When compared with a corresponding linear programming approach, it was found to be an optimizing method.[14] In addition, it has been used for more extended analyses of the layout problem.[15]

Davies, Gabbard, and Reinholdt compared four layout strategies including the COI method.[16]

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[13] J. L. Heskett, "Cube-per-Order Index—A Key to Warehouse Stock Location," *Transportation and Distribution Management*, Vol. 3 (April 1963), pp. 27–31; and J. L. Heskett, "Putting the Cube-per-Order Index to Work in Warehouse Layout," *Transportation and Distribution Management*, Vol. 4 (August 1964), pp. 23–30.

[14] Carl Kallina and Jeffery Lynn, "Application of the Cube-per-Order Index Rule for Stock Location in a Distribution Warehouse," *Interfaces*, Vol. 7, No. 1 (November 1976), pp. 37–46. See also Hoyt G. Wilson, "Order Quantity, Product Popularity, and the Location of Stock in a Warehouse," *AIIE Transactions*, Vol. 9, no. 3 (September 1977), pp. 230–237.

[15] Charles J. Malmborg and Stuart J. Deutsch, "A Stock Location Model for Dual Address Order Picking Systems," *IIE Transactions*, Vol. 20, No. 1 (March 1988), pp. 44–52.

[16] Arthur L. Davies, Michael C. Gabbard, and Ernst F. Reinholdt, "Storage Method Saves Space and Labor in Open-Package-Area Picking Operations," *Industrial Engineering* (June 1983), pp. 68–74.

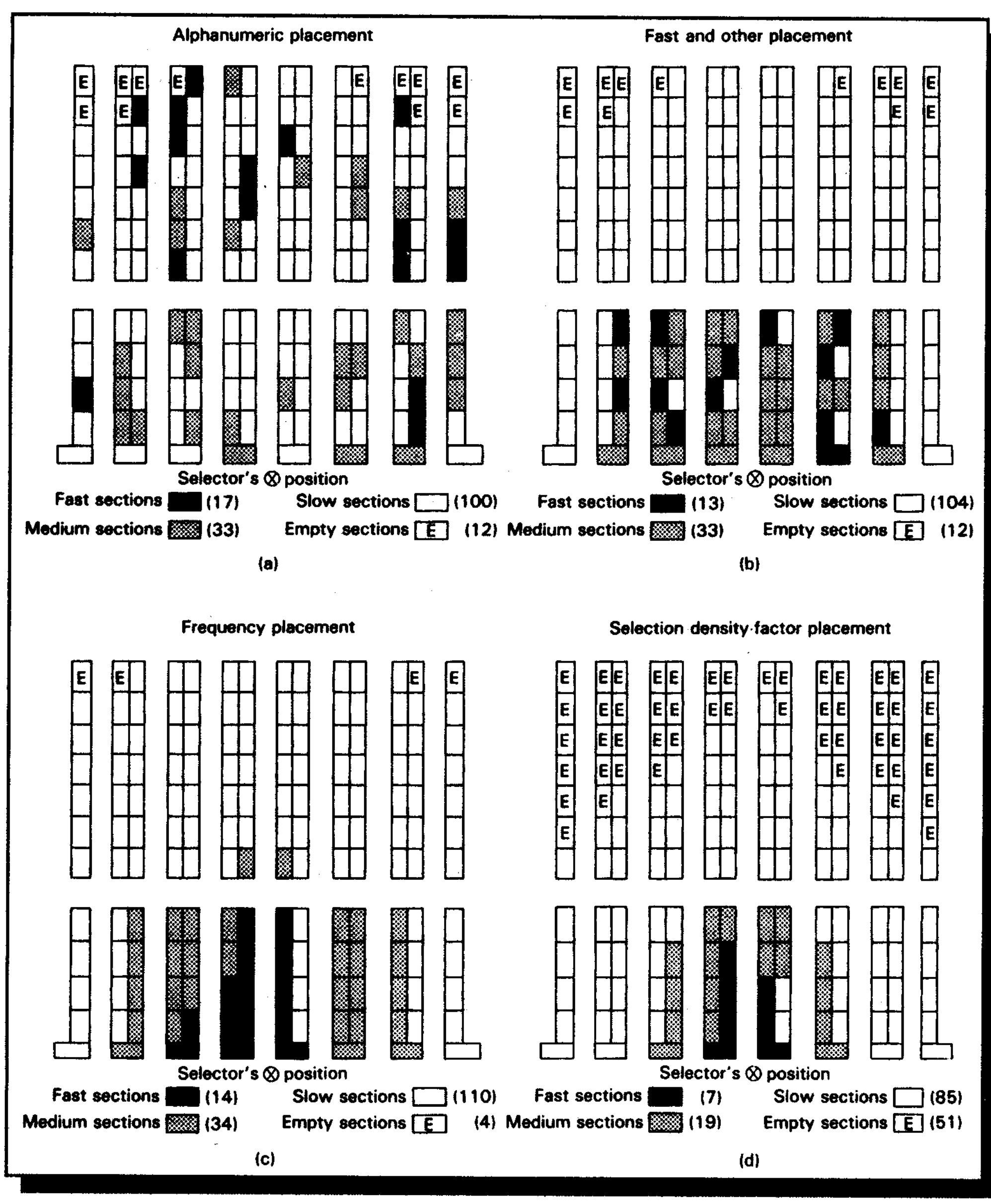


Figure 12-5 Comparison of Four Stock Location Strategies

Source: Arthur L. Davies, Michael C. Gabbard, and Ernst F. Reinholdt, "Storage Method Saves Space and Labor in Open-Package-Area Picking Operations," *Industrial Engineering* (June 1983), p. 70. Copyright Institute of Industrial Engineers, Norcross, GA.

1. *Alphanumeric placement*—all items are placed in strict alphanumeric sequence.
2. *Fast and other placement*—selected items are segregated from the remaining, or "other," items and stored in alphanumeric sequence as close as possible to the selector's work position.
3. *Frequency placement*—the fastest moving items are placed as close as possible to the selector's work position. (*Note:* This is the same as the layout-by-popularity method.)
4. *Selection density factor (SDF) placement*—the higher the ratio of number of selections per year to the required storage volume in cubic feet, the closer the item is placed to the selector's work position. (*Note:* This is the inverse of the cube-per-order index.)

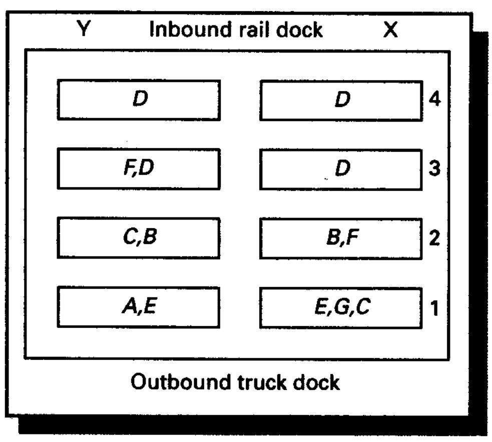
A simulation study was conducted of 800 stock items that had an average of 800 selections per day. They found SDF, or COI, placement superior to the others, as shown graphically in Figure 12-5. It produced (1) the lowest average distance per selecting trip; (2) the lowest average time per selecting trip; (3) the lowest time per line items selected; and (4) the least total space. The SDF placement method has been widely implemented at Western Electric's material distribution warehouses.

Example

A warehouse has the internal configuration shown in Figure 12-6. Each storage bay can accommodate 40,000 cubic feet of product. Data have been collected on the cubic footage of storage required for the smallest shipping unit of the item for which an order can be placed, the expected number of orders on which the item appears over the planning horizon of one year, and the expected number of units to be shipped throughout the year. The basic data for seven items are shown in Table 12-8, as well as the computation of the COI for each item. Assigning the items with the lowest COI to storage bays nearest the outbound dock leads to the following acceptable product layout:

Bay No.	Product	Percentage of Bay Capacity Used
1Y	A—4,800 cu. ft.	
	E—35,200 cu. ft.	100%
1Z	E—2,400 cu. ft.	
	G—13,600 cu. ft.	
	C—24,000 cu. ft.	100
2Y	C—1,120 cu. ft.	
	B—38,880 cu. ft.	100
2Z	B—25,120 cu. ft.	
	F—14,880 cu. ft.	100
3Y	F—4,800 cu. ft.	
	D—35,200 cu. ft.	100
3Z	D—40,000 cu. ft.	100
4Y	D—40,000 cu. ft.	100
4Z	D—33,600 cu. ft.	84

Figure 12-6 Internal Warehouse Storage Bay Structure for Example Problem with a Cube-Per-Order Index Layout



Intuitive layout methods are simple to use but do not guarantee that the lowest cost materials handling layout pattern will be found. For example, the methods just described best relate to order picking when it is of the out-and-back type. When a picker router is involved, methods for vehicle routing (see Chapter 7) are more appropriate.[17] Also, various methods that have been developed for plant layout are useful for warehouse layout as well. One such well-known model is computerized relative allocation of facilities technique (CRAFT)[18] and its various spin-off versions.[19] Computerized facilities design (COFAD) not only minimizes the movement cost but

Table 12-8 Cube-Per-Order Index Computations for Example Problem

Product	(1) Item Size, cu. ft.	(2) Expected Number of Orders/Year	(3) Average Inventory Units	(4) = (2)/250 Average Number of Daily Orders[a]	(5) = (1) × (3) Required Storage Space, cu. ft.	(6) = (5)/(4) Cube-Per-Order Index
A	6.0	6,750	800	27	4,800	177.8
B	4.0	15,750	16,000	63	64,000	1015.9
C	1.0	11,250	25,120	45	25,120	558.2
D	8.0	25,500	18,600	102	148,800	1458.8
E	3.0	17,750	12,533	71	37,599	529.6
F	5.0	3,500	3,936	14	19,680	1405.7
G	15.0	6,250	907	25	13,605	544.2
Totals		86,750	77,896		313,604	

[a]Based on 250 selling days per year

[17]See also James A. Chisman, "The Clustered Traveling Salesman Problem," *Computers and Operations Research*, Vol. 2, No. 2 (September 1975), pp. 115–119; and Marc Goetschalckx and H. Donald Ratliff, "Order Picking in an Aisle," *IIE Transactions*, Vol. 20, No. 1 (March 1988), pp. 53–62.

[18]Elwood S. Buffa, Gordon C. Armour, and Thomas E. Vollman, "Allocating Facilities with CRAFT," *Harvard Business Review*, Vol. 42 (March–April 1964), pp. 136–158.

[19]R. L. Francis and J. A. White, *Facility Layout and Location: An Analytical Approach* (Upper Saddle River, NJ: Prentice Hall, 1974).

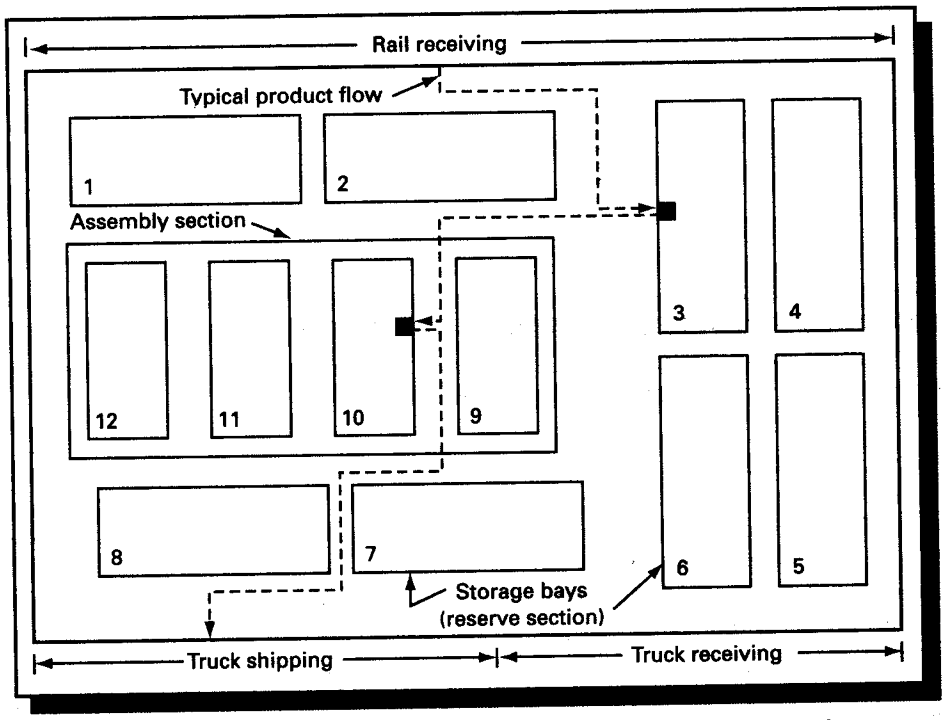


Figure 12-7 Sample Arrangement of Reserve and Assembly Areas in Grocery Warehouse

also assigns materials handling equipment to given types of moves.[20] SPACECRAFT extends the CRAFT model to multistory facilities by appending additional floors to a first floor.[21] In an interesting comparative study, Trybus and Hopkins found that computer methods (specifically, CRAFT) gave better results than human subjects could find as problem size increased.[22] CRAFT always did as well, regardless of problem size. MULTIPLE extends CRAFT to multiple stories and realizes improved solutions from using space filling curve technology.[23] Now, even the expert systems approach is being applied to the layout problem.[24]

A more complex problem is two-stage layout, as shown in Figure 12-7. Product is received at rail or truck docks and is moved to semipermanent (reserve) storage. As

[20]"COFAD—A New Approach to Computerized Layout," *Modern Materials Handling* (April 1975), pp. 40–43.
[21]Roger V. Johnson, "Spacecraft for Multi-Floor Layout Planning," *Management Science*, Vol. 28, No. 4 (April 1982), pp. 407–417.
[22]Thomas W. Trybus and Lewis D. Hopkins, "Humans vs. Computer Algorithms for the Plant Layout Problem," *Management Science*, Vol. 26, No. 6 (June 1980), pp. 570–574.
[23]Yavuz A. Bozer, Russell D. Meller, and Steven J. Erlebacher, "An Improvement-type Layout Algorithm for Single and Multiple-floor Facilities," *Management Science*, Vol. 40, No. 7 (July 1994), pp. 918–932.
[24]John G. Carlson and Andrew C. Yao, "A Visually Interactive Expert System for a Distribution Center Environment," *International Journal of Production Economics*, Vol. 45, No. 1 (August 1, 1996), pp. 101–109.

Product	Mode of Delivery	Turnover Ratio	Space Requirements: Warehouse	Space Requirements: Assembly[a]	Bay Capacities: Reserve	Bay Capacities: Assembly
1	Rail	15	9,300	62	5,000	2,500
2	Truck	14	1,600	18	1,000	500
3	Truck	17	3,800	69	4,000	2,000
4	Rail	16	5,700	96	2,000	1,000
5	Rail	20	18,000	160	8,000	4,000

[a]These are minimum requirements for the assembly.

Table 12-9 Storage Bay Capacities and Space Requirements in Units for the Reserve and Assembly Areas for the Sample Grocery Warehouse

stock is depleted in the order-picking (assembly) area, replenishment stock is moved from the storage section to the order-picking section. As orders are filled, product is moved from the order-picking section to the outbound dock. The questions are where to place each product in the warehouse and how much space should be allocated for each product in the semipermanent and order picking sections. Table 12-9 illustrates a hypothetical example of this problem using only a few products and data to show contrasts.

A linear programming model can be formulated as an approach to this problem. It is shown in the Technical Supplement to this chapter. What we wish to do is minimize the total costs of moving the products through the warehouse, subject to limitations on minimum amounts of product to be stored in the assembly section, in a particular bay, and in the warehouse. Since products cannot occupy the same locations, this becomes an allocation problem to be solved. Once the per unit handling costs are estimated for the various product flow paths through the warehouse, the problem can be easily solved by most general-purpose linear programming computer programs. Although not all data for this problem are given here, the general nature of the solution would be as shown in Table 12-10.

Conceptually, linear programming is a good choice for solving the layout problem because, in effect, all possible arrangements are searched to find an optimum, and the assembly and reserve sections can be laid out simultaneously. However, practical problems involving thousands of products may be too large to be reasonably solved by linear programming. Therefore, application of methods discussed in this section, especially those developed for plant layout, may require creating zones of product in the warehouse or grouping of products into families to limit problem size. In addition, a method such as CRAFT achieves a higher computational speed than linear programming with little loss in solution accuracy. As Buffa, Armour, and Vollman point out: "The answers generated are not as surely the best ones as the answers to linear programming problems are, but they do represent solutions that cannot easily be improved on."[25]

[25]Buffa, Armour, and Vollman, "Allocating Facilities with CRAFT."

Bay	Product 1	2	3	4	5[a]
1	4,238			305	
2	5,000				
3		5		1,190	
4		510			
5		1,000			
6		67	3,371		
7				1,309	2,765
8				2,000	
9		18		96	3,472
10					4,000
11	62				4,000
12			69		3,763
Total requirements	9,300	1,600	3,800	5,700	18,000

[a]Much of product 5 is located in the assembly section, due to the high product turnover. If this creates too much of an imbalance with the other products, either the space requirements on products 1 to 4 can be increased, or a constraint can be added to the model that will limit the amount of a product that should be stored in the assembly section.

Table 12-10 Amount of Each Product Assigned to Respective Bays to Achieve Minimum Total Handling Cost for a Grocery Warehouse

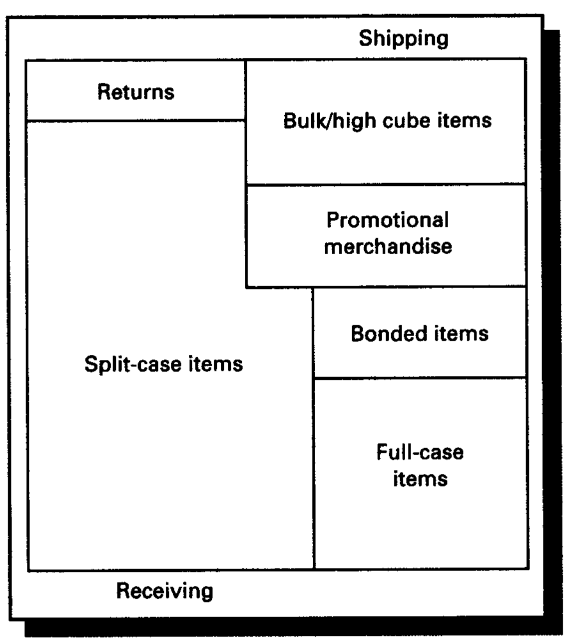
Activity Profiling

A warehouse is not typically an area in which a singular layout occurs. Rather, the area is frequently divided into several subareas with specialized functions. Depending on the activity level and product mix, defined areas can be (1) full pallet/full case, (2) split case, (3) bulk, (4) bonded, (5) promotional, (6) returned merchandise, and (7) administrative. To determine the need for these areas and their size, Frazelle suggests a data mining process referred to as *activity profiling*.[26] Statistical distributions are obtained from actual sales data on order mix, lines per order, cube per order, and lines-and-cube per order. These data are also useful for applying the stock location methodology described in the previous section.

A first step in activity profiling is to generate an order mix distribution. We seek to find how much of the order volume is in pallet-load, full case, and broken-case quantities. Since stock retrieval is distinctly different for these three areas in both storage configuration and handling procedures, sampling warehouse throughput for a reasonable time, say, one year, provides the activity level needed to design these areas. Other merchandise classified as bulk, bonded, and promotional can be handled in a similar manner. Apportioning the warehouse space to the various uses might lead to the space allocation shown in Figure 12-8 for a high-volume warehouse.

[26]Edward Frazelle, *World-Class Warehousing and Material Handling* (New York: McGraw-Hill, 2002), Chapter 2.

Figure 12-8
Area Configuration for a High Throughput Warehouse Based on Activity Profiling

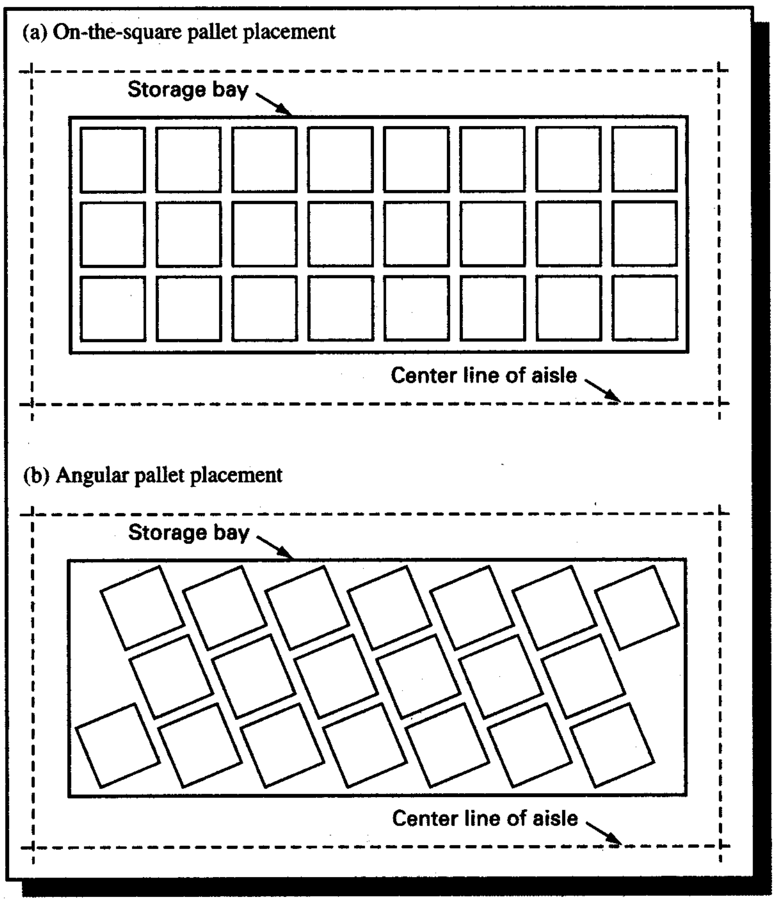


The same order data are subdivided along area lines. For example, merchandise assigned to a split-case picking area is separated from the remaining data. From these data, items can be ranked by number of orders on which the item appears (representing order-picking trips) and by item size. Inventory held for each item is also obtained, but not from the sales data. These data can then be used to calculate layout assignments by popularity, by cube, or by cube-per-order index.

Another distribution would be that of demand correlation. Here, the items most often ordered together are ranked from the highest frequency of occurrence to the lowest. This establishes item complementarity suggesting those items that should be located adjacent to each other. Similarly, the data may be analyzed for seasonal patterns with the purpose of locating items within the same zone that have opposing seasonal patterns. By colocating these items, storage space requirements are reduced.

Not all areas of the warehouse require activity profiling. Once the overall size of the area has been established from a first-level statistical analysis, layout may be a matter of convenience and good judgment. The area for promotional merchandise has goods purchased in quantities exceeding normal replenishment amounts that are temporarily stored until sales from aggressive pricing deplete them. The uncertain nature of the goods and their space requirements within the promotional area suggests that systematic layout planning is not required. Activity profiling is most beneficial where handling costs are high, many items are to be stored, and there are substantial differences in the physical characteristics of the items.

Figure 12-9
Pallet Positioning Alternatives



Stock Arrangement

Efficiency in warehousing can also be enhanced by stock positioning within storage bays. Positioning is a major consideration where palletized storage is used, and palletization of goods is a common practice in many warehouse operations.

Positioning specifically refers to the angle at which pallets are laid out relative to the service aisle. The most widely used positioning is the on-the-square, or 0 degrees, placement angle Figure 12-9(a). Most warehouse operators prefer the on-the-square pallet positioning. As an alternative, pallets may be positioned at some angle to the centerline of the service aisle Figure 12-9(b). "Angling" is not used a great deal by warehouses, probably because of the continuing controversy as to whether any efficiency results from angular positioning. The controversy can be seen in those studies that have suggested angles from 0 to 60 degrees as best.[27] More important than a generally suggested

[27]Joseph J. Moder and Herbert M. Thorton, "Quantitative Analysis of Factors Affecting Floor Space Utilization of Palletized Storage," *Journal of Industrial Engineering*, Vol. 16 (January-February 1965), pp. 8–18; Donald J. Bowersox "Resolving the Pallet Controversy," *Transportation and Distribution Management* (April 1963), pp. 27–31; and Ronald H. Ballou, "The Consideration of Angular Pallet Layout to Optimize Warehouse Space Utilization" (master's thesis, The Ohio State University, 1963).

angle are the issues involved in using angular positioning and how the correct angle, whether 0 degrees or not, can be determined.

Opponents to angling complain that unused space is created at the front, back, and sides of the bay [see Figure 12-9(b)]; that column arrangement, building configuration, and floor area place limitations on the implementation of an angular positioning plan; that angled pallets are more difficult to spot in the bay at the correct angle; and that the one-way aisles that naturally result contribute to higher materials handling costs. Proponents of angling, on the other hand, argue that the reduction in aisle width due to the less-than-90-degree turn required of a forklift truck servicing a pallet more than offsets the unused space in the storage bay. In addition, some operating efficiency is gained because the forklift truck makes less than a full 90-degree turn to place or retrieve a pallet.

Resolving this controversy is primarily a matter of balancing space utilization considerations against materials handling efficiencies. The effect of angling on total space requirements has been examined, and formulas or computation forms are available for determining the exact angle for any combination of pallet size, bay configuration, and forklift truck.[28] The effect of angling on operating efficiency can be determined from a time study of forklift truck operations under different pallet angles. Converting space and time measures to economic terms, we say that the angle yielding the lowest cost can be found.

Stock Locator-Identification Methods

An important design consideration that can substantially affect materials handling efficiency is the method used to identify merchandise location in the storage bays. Two opposing identification and location schemes are the fixed locator and the random locator methods.

Consider a common location problem. When goods arrive at the warehouse, they must be placed in a storage bay somewhere in the warehouse. When an order is to be filled, the appropriate goods must be found and retrieved from the storage location. How can this be accomplished with efficiency when the existing products show increasing and decreasing stock levels due to variations of supply and demand and when the product mix is changing because of additions and deletions from the product line?

The *fixed locator-identification method* assigns a given storage bay or storage rack number to each product. These locations can be determined from the stock location methods (by popularity, COI, and the like) already discussed. This locator-identification method is simple, and no formal code is needed to identify locations if only a few items are stored in the warehouse. Personnel who place and retrieve the stock can simply memorize the locations. If the product line is extensive, a formal code can be created to identify warehouse section, bay number, and slot.

The primary disadvantage of this method is that much underutilized space may be created. Space capacity should be established for the peak stocking requirements

[28]See Moder and Thorton, "Quantitative Analysis of Factors Affecting Space Utilization of Palletized Storage"; and Ronald H. Ballou, "Pallet Layout for Optimum Space Utilization," *Transportation and Distribution Management* (February 1964), pp. 24–33.

of each product. Because product peak stocking levels usually do not occur simultaneously, poor utilization of the space can result.

The *random locator-identification method* is designed to overcome the disadvantage of the fixed locator-identification method. When goods arrive at the warehouse, they are routed to any open space that is available. There are no preassigned locations. This method offers better use of available storage space, but to keep track of many items, when each may be located in several places, requires an effective retrieval code. Because of a continually shifting space availability pattern in the warehouse, an elaborate manual or computer-based stock-filing system is needed to support this manner of operation.

Although the random locator-identification method offers improved space utilization, longer travel times are usually encountered because a single item on an order may require picking from several locations. This method, and modified versions of it, has been popular in automated storage and retrieval systems, where space costs are high relative to handling costs.

In high-volume and palletized handling systems, a blend of the two methods has proved practical. A popular modification is to confine items to designated zones in the warehouse, as suggested by stock location methods. Within these zones, products can be stored on a space-available basis.

Example

A steel distributor put the designated-zones idea to good use to store its finished coils and sheet goods. The storage area was divided into a number of smaller areas that were given color designators—pink, violet, orange, and so on. The colors provided easy identification of each area and of the product within. The product stored in a given area was allowed to "float" without a specifically assigned spot. Although the product could move randomly within its designated area, the area was not so large that product would be easily lost yet provided good utilization of the space.

Order-Picking Operations

The labor-intensive nature of order picking makes it a target for productivity improvements. Several operational considerations can improve materials handling efficiency.

Order Handling

How the incoming order is managed affects handling costs. Generating picker lists from the sales order can lower costs.

Product Sequencing

Sequencing is the arrangement of items on picking route lists so that they are picked in an efficient route through the stock. Order-picking time is saved by avoiding backtracking through aisles and merchandise. Sequencing the items as they occur on the sales order may require the cooperation of sales personnel and customers to list items in the designated order. Alternately, a popular approach is the use of computers to sequence sales order items into efficient picker lists.

Picker Zoning

Zoning refers to assigning individual order pickers to serve only a limited number of the stock items instead of routing them through the entire stock layout. An order picker selects stock only within a designated area and usually fills only a portion of the total customer order. To achieve low materials handling cost (reduce picker fatigue and maximize throughput), careful attention needs to be paid to several factors. First, stock should be located between and within picker zones according to order frequency, complementarity, item weight, rack position, and item cube so that order picker workloads between zones are balanced. Second, the sales order must be broken down into picker lists for each zone. Third, the various portions of the order must be assembled into a complete order before leaving the warehouse. If the order filling proceeds sequentially from one zone to another to avoid the problem of reassembly when the zones are dispersed, then the order-picking pace becomes dependent upon the pace of order picking in other zones.

Although picker zoning has been the popular approach to dividing workload in high throughput warehouses, an alternative philosophy is emerging. It is derived from "swarm intelligence," the collective behavior of social insects such as ants, bees, and wasps.[29] Watching how ants move food from source to nest, the "bucket brigade" approach is observed. That is, ants pass food from one member to another along a food gathering chain. The ants are not stationary and the transfer points are not fixed. Starting with the food source, an ant carries the food down the chain until it reaches the next ant. After transferring the food, it returns upstream until it reaches the previous ant to receive the next load. This process continues along a chain of multiple ants where the only fixed locations are the food source and the nest.

Swarm intelligence has been applied to order picking in large distribution centers where a 31 percent improvement over the zone approach has been reported.[30] The zone approach does not recognize the wide variation in rate at which the pickers complete their tasks. The quickest person could be four times faster than the slowest. This tends to underutilize the fast people and aggravate the slower ones who are under pressure to keep up. Even if all worked at the same rate, the normal variation in completing the tasks in each zone would make it difficult to balance the workload. The better approach is for an upstream worker in an order-filling chain to continue picking items on the order until the person downstream takes over the work; then, head back upstream to take over the next person's work. The optimal way to arrange the workers is to begin upstream and sequence them from *slowest to fastest*.

[29]Eric Bonabeau and Christopher Meyer, "Swarm Intelligence: A Whole New Way to Think About Business," *Harvard Business Review*, Vol. 79, No. 5 (May 2001), pp. 106–114.
[30]Ibid.

Order Splitting

Order splitting is an extension of the ideas of picker zoning. When the stock does not reside at a single location, it is necessary to divide the sales order before routing it to a warehouse.

Application

Rico Drug Stores receives weekly replenishment orders from its retail stores. The orders are first split between over-the-counter (OTC) merchandise and pharmaceuticals. Pharmaceuticals are stocked at one location in the country. The remainder of the order is sent to the local distribution center where OTC merchandise is stocked. The order is further divided between bulk merchandise, which is stocked at a public warehouse, and the remainder, which is held in a leased facility. The flow of merchandise is coordinated from these separate locations so that the order arrives at the retail store at the promised time. Tagging and labeling the items and computer tracking of the split-order items become critical to achieving overall order coordination.

Item Batching

Batching is the selection of more than one order on a single pass through the stock. This practice obviously reduces travel time, but it also adds to the complication of reassembling orders and partial orders for shipment. It also may increase order-filling time for any order because its completion is dependent upon the number and size of the other orders in the batch.

Interleaving

A special problem in order-picking operations occurs when storage and order picking take place at the same time on the same route from the same origin-destination point. This has been referred to as *interleaving* and it is a common problem found in automatic storage and retrieval systems. For random storage assignment, where any open rack is selected for storage, a common rule is to select the open location nearest to the origin-destination point. However, a storage-retrieval rule based on turnover (popularity) has been shown to reduce substantially the average trip time for storage alone or for interleaving.[31]

Setting Standards

High levels of materials handling efficiency cannot be guaranteed by the application of rules, concepts, or optimization methods alone. The worker is an important ingredient

[31] Leroy B. Schwarz, Stephen C. Graves, and Warren H. Hausman, "Scheduling Policies for Automatic Warehousing Systems: Simulation Results," *AIIE Transactions*, Vol. 10, No. 3 (September 1978), pp. 260–270.

in the total cost equation. Performance standards are important to providing norms so that a reasonable number of workers can be assigned to the work of warehousing, to provide a benchmark against which superior or substandard performance can be judged, and to provide a base wage for incentive systems so that increased productivity can be rewarded.

Concluding Comments

This chapter deals with planning the design and operation of storage facilities with emphasis on the warehouse. Logisticians will have varying needs for this material, depending on how storage is provided in their firms. If public warehousing is used, the managers of the public warehouses plan the operation and the user firms evaluate the rates and services on a comparative basis with other public warehousing firms. On the other end of the scale, if the storage space is to be company owned, the logisticians will face the full range of warehouse design and operations decisions.

The discussion focuses on the various planning decisions relating to the major space and materials handling problems, once the general warehouse location is known. These major decisions include structure size and financial arrangement, facility configuration, space layout, dock design, materials handling systems selection, equipment replacement, stock arrangement, stock locator-identification methods, and order-picking operations. Concepts as well as mathematical models for decision making are illustrated. Although storage and materials handling decisions are presented here as seemingly independent of each other, and of the logistics system as a whole, the logistician is cautioned to watch for the economic impact that each of the noted warehouse decision problems has on other decisions outside their immediate scope. Activity profiling is suggested as a means for providing the initial information needed for warehouse design.

Questions

Some problems in this chapter can be solved or partially solved with the aid of computer software. The software packages in LOGWARE that are most important for this chapter are LNPROG (*LP*) and LAYOUT (*LO*). The CD icon will appear with the software package designation where the problem analysis is assisted by one of these software programs. A database may be prepared for the problem if extensive data input is required. Where the problem can be solved without the aid of the computer (by hand), the hand icon is shown. If no icon appears, hand calculation is assumed.

1. A warehouse is to be located somewhere in your hometown. What factors do you think should be evaluated in making the selection of a particular site?

2. The Acme Manufacturing Company is concerned about its warehouse needs and how they can best be met. The company produces a line of spare parts for appliances. Due to the combination of production policies and demand patterns, warehousing space requirements vary considerably throughout the year. Space requirements are known with a great deal of certainty because the product line satisfies a replacement market. Growth, or decline, in production and sales is not anticipated in the near future. Monthly sales rates for a typical year are as follows:

Month	Sales, $
Jan.	5,000,000
Feb.	4,000,000
Mar.	3,000,000
Apr.	2,000,000
May	1,000,000
June	250,000
July	1,250,000
Aug.	2,250,000
Sept.	3,000,000
Oct.	3,500,000
Nov.	4,000,000
Dec.	4,500,000
Total	33,750,000

Warehouse inventory turns at the rate of two times per *month*. A dollar's worth of merchandise occupies 0.1 cubic feet of warehouse space and can be stacked 10 ft. high. The product density is $5 per lb. Given aisles, administrative space, and normal operating efficiency, only 40 percent of the total warehouse space is actually used for storage.

A private warehouse can be constructed and equipped for $35 per sq. ft. and can be amortized over 20 years. The cost of operation is $0.02 per dollar of throughput (lb). Annual fixed costs amount to $10 per sq. ft. of total space. Space may also be rented for a storage charge on inventory of $0.06 per lb per month and a handling charge of $0.05 per lb of throughput.

What size of private warehouse should be constructed, to the nearest 10,000 sq. ft., or what amount of public warehouse space should be rented? To what extent and when should each type of space be used?

3. O'Neal Consumer Products is in need of 150,000 sq. ft. of warehouse space for its East Coast market, where annual sales are $30,000,000. If a public warehouse is used, annual costs can be estimated as $600,000 for handling and $300,000 for storage. If leased space is used, the annual lease cost is $3 per sq. ft. for a ten-year lease. Leased-space operating cost is $250,000 per year. Equipment and start-up costs are $400,000, which can be depreciated over a seven-year period. A straight-line depreciation schedule is to be used.

The company's required return on projects is 11 percent after taxes, and its federal tax rate is 35 percent per year.

Which alternative makes the best economic sense?

4. A private warehouse has an annual throughput of 10,000 items and an average materials handling cost per item of \$0.01/ft. The warehouse size is to be 100,000 sq. ft. Annual construction and maintenance costs are \$210/ft. of perimeter. The loading and unloading dock is to be located at a warehouse corner. What is the best length and width of the warehouse? What is the total relevant cost for this design?
5. Using the data given in the space layout example from the chapter, design the layout and the length and width dimensions of a warehouse in the style of Figure 12-4(a).
6. A food distribution center makes deliveries to food stores on a weekly basis. On the average, 75 stores are served daily. A typical store places an order for 12,000 lb of various products. Three store orders can be placed on a delivery truck. Trucks are loaded in three hours. The distribution center operates an eight-hour shift.

   How many truck doors are needed on the average?
7. A firm uses a number of narrow-aisle forklift trucks and can purchase these in three types. Type 1 costs \$20,000 each; type 2 costs \$10,000 each; and type 3 costs \$5,000 each. Such equipment can be sold at the end of its useful life (ten years) for 15 percent of its original cost. The annual operating costs for each type of equipment are \$2,000, \$2,500, and \$3,000, respectively per truck. Three type-1 units can do the work of five type-2 units or of seven type-3 units. If investments are to return 20 percent before taxes per truck, which equipment would be the best buy?
8. A certain narrow-aisle forklift truck costs \$4,000. When it is replaced, it will be replaced with a truck of the same kind. Operating costs for this truck are \$500 for the first year and increase at the rate of \$40 per year *squared* thereafter. Technological improvements reduce operating costs by an estimated \$30 per year. The salvage value of the trucks declines linearly over their seven-year life. The desired rate of return is to be 20 percent before taxes.

   When should the equipment be replaced?
9. Suppose that a warehouse contains eight storage bays. Product enters the rear of the warehouse through a rail dock. Product is picked from the storage locations by an out-and-back selection method and shipped from a truck dock at the front of the building (see the design in Figure 12-6). Each bay can hold 2,500 sq. ft. with product stacked 10 ft. high. Ten products are maintained in the warehouse. The following data have been collected:

Product	Storage Space Required (sq. ft.)	Individual Product Size (cu. ft.)	Average Number of Daily Orders in Which Item Appears
A	500	1.5	56
B	3,000	10.6	103
C	1,500	4.3	27
D	1,700	5.5	15
E	5,500	2.7	84
F	1,100	15.0	55
G	700	9.0	26
H	2,800	6.7	45
I	1,300	3.3	94
J	900	4.7	35

a. Lay out the warehouse using the (1) by-popularity method, (2) the by-cube method, and (3) cube-per-order index method.
b. To what extent are these methods appropriate when more than one item is picked on a route, and when pickers are zoned to pick only a limited part of the product line on each order?

10. The Able Company is a local division of a large public warehousing firm. The management of this company has successfully applied the techniques of scientific management in the past and is currently looking at its layout problem to see if these techniques can indicate whether cost savings can be made in this area. The company has selected a particular warehouse for consideration. This warehouse has two receiving docks (R_1, R_2) and one shipping dock (S_1). The three major products handled by the warehouse are stored in six storage bays.

Management finds that because of order sizes, receiving locations, quantities received, and the like, different times are required to supply and distribute from a storage bay, and these service times depend on the particular product and location of the storage bay in the warehouse. There is a direct relationship between handling costs and handling times for each product and storage bay.

HANDLING TIMES (HR.)[a] PER 100 UNITS OF PRODUCT STORED IN VARIOUS BAYS

Storage Bay	1	2	3
1	0.90	0.75	0.90
2	0.80	0.65	0.95
3	0.60	0.70	0.65
4	0.70	0.55	0.45
5	0.50	0.50	0.45
6	0.40	0.45	0.35

[a]For a three-month period

Each storage bay has a certain capacity depending on the product. The following information on storage bay capacity is known:

Product	Storage Bay Capacity (units)
1	5,000
2	3,000
3	6,000

Management forecasts that it must plan storage space for at least 11,000 units of product 1, 4,000 units of product 2, and 12,000 units of product 3 over the next three months. The decision problem is how to allocate the products to the various storage bays (in the proper quantities) so as to minimize the total handling time (cost) required for all products. (*Hint*: Solve as a linear programming problem using the following model):

Objective function

$$z_{\min} = \sum_i \sum_j C_{ij} X_{ij}$$

subject to

$$\sum_j \frac{1}{G_j} X_{ij} \leq 1.0 \quad \text{for } i = 1, 2, \ldots, M$$

and

$$\sum_i X_{ij} \geq R_j \quad \text{for } j = 1, 2 \ldots, N$$

where

G_j = capacity of bay for product j
R_j = number of units of product j required to be stored
M = number of storage locations
N = number of products

11. What space trade-offs are involved in angular pallet positioning? What additional considerations would enter into the decision to use angular pallet positioning?
12. What alternative methods of stock location and retrieval can you think of? Discuss the advantages and disadvantages of the methods that you propose.
13. A leading manufacturer of rubber and vinyl houseware products uses a random stock locator-retrieval system in its plant warehouse. All orders in the country are filled through this location. The internal warehouse design shows seven-tier racks laid out in rectangular patterns. The materials handling system involves narrow-aisle forklift trucks and palletized storage. Why would this company likely find such a storage materials handling system an advantage over other types?
14. A parts warehouse has two types of storage areas. The first type is carousels having many bins in which small and frequently requested items are placed. The remaining items are placed on storage racks (second type) from which items are retrieved using forklift trucks. What data distributions would you construct and how you would use them (activity profiling) to determine the size of the carousel/storage rack space? Next, how would the data distribution be used to lay out the items within theses areas?
15. For order picking in a warehouse, contrast the zone approach with the bucket brigade approach.

Appendix

Technical Supplement

The general linear programming formulation for the product layout problem involving both reserve storage and order picking areas is as follows:

The objective is to minimize the total materials handling cost, that is,

$$z_{\min} = \sum_{i=1}^{M} \sum_{j=1}^{N} C_{ij} X_{ij}$$

subject to

1. a reserve section bay capacity constraint:

$$\sum_{j=1}^{N} \frac{1}{G_j^s} X_{ij} \le 1.0 \quad \text{for } i = 1, 2, \ldots, L$$

2. an assembly section bay capacity constraint:

$$\sum_{j=1}^{N} \frac{1}{G_j^a} X_{ij} \le 1.0 \quad \text{for}$$
$$i = L + 1, L + 2, \ldots, M$$

3. the minimum number of units of each product to be stored in the assembly section:

$$\sum_{i=L+1}^{M} X_{ij} \ge R_j^a \quad \text{for } j = 1, 2, \ldots, N$$

4. the total number of units to be stored throughout the warehouse:

$$\sum_{i=1}^{M} X_{ij} \ge R_j \quad \text{for } j = 1, 2, \ldots, N$$

5. a negative amount of product *j* cannot be stored:

$$\text{all } X_{ij} \ge 0$$

where

X_{ij} = amount of product j stored in bay i
C_{ij} = cost for handling product j when stored in bay i
M = number of storage bays in both reserve and assembly sections
N = number of different stock items handled by the warehouse
L = number of storage bays in the reserve section
G_j = amount of product j that can be stored in a bay
R_j = the required amount of product j to be stored in the warehouse
R_j^a = the minimum amount of product j to be stored in the assembly section

s and a = superscripts to denote the reserve and assembly sections, respectively

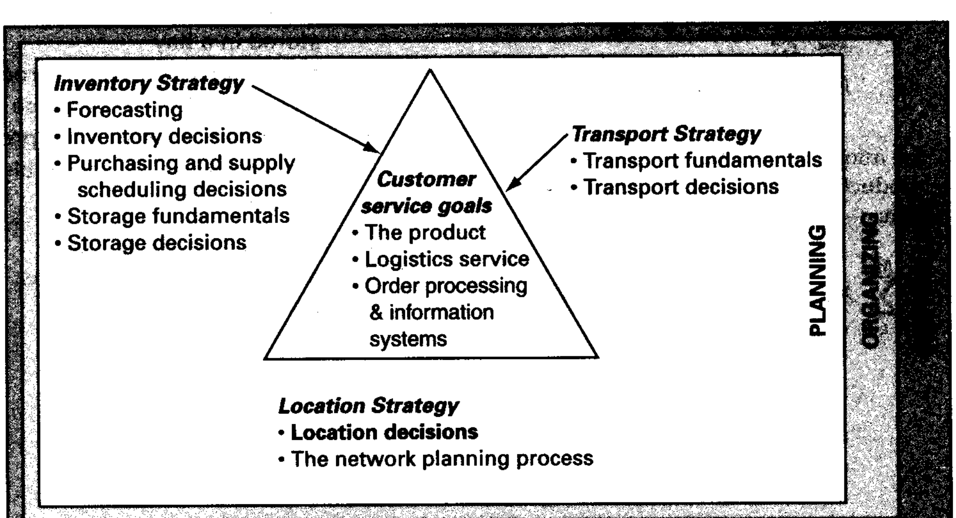
PART V

Chapter 13

Facility Location Decisions

> ... what they ... and most ... occupations are ... productions.
>
> —ALEXANDER HAMILTON, 1791

Locating fixed facilities throughout the supply chain network is an important decision problem that gives form, structure, and shape to the entire supply chain system. This design defines the alternatives, along with their associated costs and investment levels, used to operate



the system. Location decisions involve determining the number, location, and size of the facilities to be used. These facilities include such nodal points in the network as plants, ports, vendors, warehouses, retail outlets, and service centers—points in the supply chain network where goods temporarily stop on their way to final consumers.

Developing methods for locating facilities has been a popular area for research.[1] In this chapter, we will look at a selected number of the available methods for strategic network planning. The focus will be on those methods that (1) are representative of the types of solution methods available; (2) address a variety of common business location problems; and (3) illustrate the issues facing the decision maker in network planning.

Classification of Location Problems

When discussing location methods, it is useful to classify location problems into a limited number of categories, namely, by (1) driving force, (2) number of facilities, (3) discreteness of the choices, (4) degree of data aggregation, and (5) time horizon.

Driving Force

Facility location is often determined by one critical factor. In the case of plant and warehouse location, economic factors usually dominate. In retail location, revenue generated by a location is often the determining factor, with site costs subtracted from revenues to determine profitability. Where a service operation (hospital, automated bank teller, charity collection center, or maintenance facility) is to be located, accessibility to the site may be the primary location factor, especially when revenue and costs are not easily determined.

Number of Facilities

Locating one facility is a considerably different problem from locating many facilities at one time. Single facility location avoids the need to consider competitive forces, division of demand among facilities, inventory consolidation effects, and facility costs. Transportation costs are typically the primary consideration. Single facility location is the simpler of the two problem types.

Discreteness of the Choices

Some methods will explore every possible location along a space continuum and select the best one. These we refer to as *continuous* location methods. Alternatively,

[1]For a survey of many of these methods, see Margaret L. Brandeau and Samuel S. Chiu, "An Overview of Representative Problems in Location Research," *Management Science*, Vol. 35, No. 6 (June 1989), pp. 645–674; and Zvi Drezner, *Facility Location* (New York: Springer-Verlag, 1995).

location methods may select from a list of possible choices that have been identified for their reasonableness. These are *discrete* location methods. The latter are more commonly used in practice, mainly for multiple facility location.

Degree of Data Aggregation

Location problems typically involve the evaluation of an exceedingly large number of network design configurations. To manage problem size and obtain a solution, it is generally necessary to use aggregate data relationships when solving a practical location problem. This results in methods whose accuracy limits locations to wide geographic areas such as entire cities. On the other hand, methods using little data aggregation, especially those for site selection, can differentiate between locations separated only by a city street. The latter is particularly needed for retail location, intracity locations, and making final plant and warehouse site selections.

Time Horizon

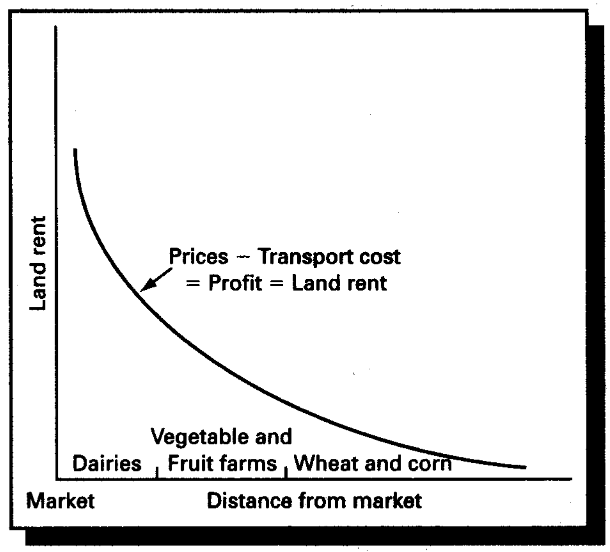
The time nature of location methods is to be static or dynamic. That is, static methods find locations based on data for a single period, such as one year. Location plans may cover many years at once, however, especially if facilities represent a fixed investment and the costs of relocating from one location to another are high. Methods that handle multiperiod location planning are referred to as dynamic.

A HISTORICAL PERSPECTIVE ON LOCATION[2]

Much of the early theories about location were postulated by land economists and regional geographers such as Johann von Thünen,[3] Alfred Weber,[4] T. Palander,[5] August Lösch,[6] Edgar Hoover,[7] Melvin Greenhut,[8] and Walter Isard.[9] A common theme throughout all of these early works was the importance of transportation costs in determining location. Although much of the work was conducted in an agrarian and early industrial society, a number of the concepts that they suggested are still applicable today. Consider a brief outline of just a few of these.

[2]For a review of the history of location modeling, see T. Puu, *Mathematical Location and Land Use Theory* (New York: Springer-Verlag, 1997).
[3]Johann Heinrich von Thünen, *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie*, 3rd ed. (Berlin: Schumacher-Zarchlin, 1875).
[4]Alfred Weber, *Uber den Standort der Industrien* (Mohr, Tubingen, 1909), translated by Carl J. Friedrich as *Alfred Weber's Theory of the Location of Industries* (Chicago: University of Chicago Press, 1929).
[5]T. Palander, *Beitrage zur Standortstheorie* (Uppsala, 1935).
[6]August Lösch, *Die Raumliche Ordnung der Wirtscaft* (Jena: Gustav Fischer Verlag, 1940).
[7]Edgar M. Hoover, *Location Theory and the Shoe and Leather Industries* (Cambridge, MA: Harvard University Press, 1957).
[8]Melvin L. Greenhut, *Plant Location in Theory and Practice* (Chapel Hill, NC: University of North Carolina Press, 1956).
[9]Walter Isard, et al., *Methods of Regional Analysis: An Introduction to Regional Science* (New York: John Wiley & Sons, 1960); and Walter Isard, *Location and Space Economy* (Cambridge, MA: MIT Press, 1968).

Figure 13-1
Thünen's Rent Curve for Land



Bid-Rent Curves

Thünen recognized that the maximum rent, or profit, that any economic development could pay for land was the difference between the price for the goods *in* the marketplace and the cost of transporting the goods *to* the marketplace. He visualized an isolated city-state (marketplace) situated on a plane of equal fertility. Economic activity would locate itself around this city-state according to its ability to pay for the land. In an agricultural economy, agricultural activity might locate out from the marketplace, as shown in Figure 13-1. Today, this idea still seems to hold as we observe the pattern of retail, residential, manufacturing, and agricultural locations that ring the city center. Those activities that can pay the most for land will be located nearest the city center and along major transportation links.

Weber's Classification of Industries

Alfred Weber recognized the role that raw materials play in the production process and how they affect location. He observed that some processes are weight losing, such as steelmaking. That is, the sum of the weight of raw materials is greater than the weight of the finished product. Weight is lost in processing due to unusable by-products. Therefore, to avoid shipping by-products to the marketplace, such processes are drawn toward their raw material sources in order to minimize transportation costs (see Figure 13-2).

On the other hand, processes may be weight gaining. This commonly occurs when ubiquities enter into the process. According to Weber, ubiquities include the raw materials available everywhere, such as air and water. Therefore, to minimize transportation costs by shipping ubiquities the shortest possible distance, such

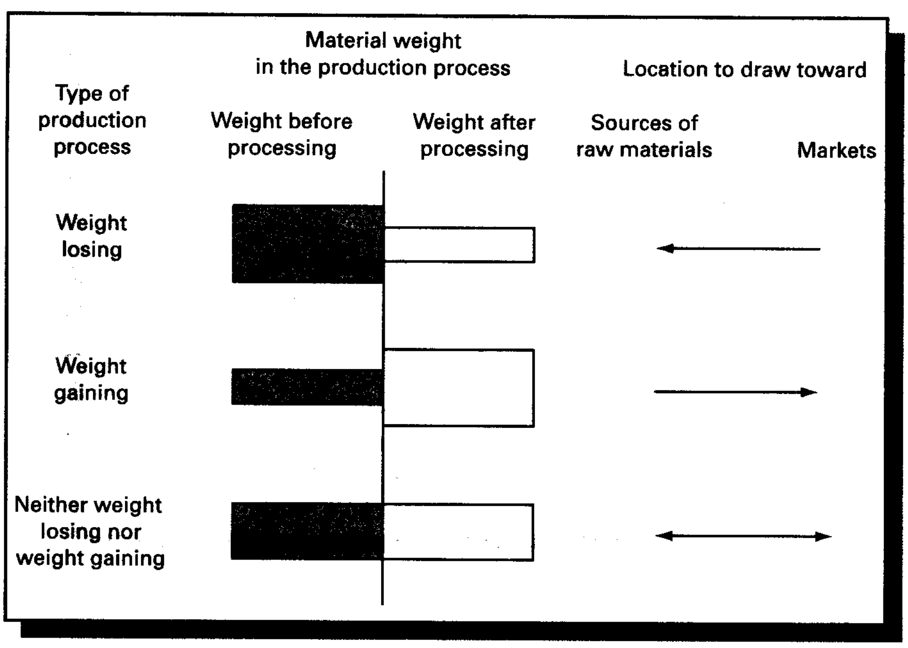


Figure 13-2 Effect on Process Location of Product Weights Before and After Processing

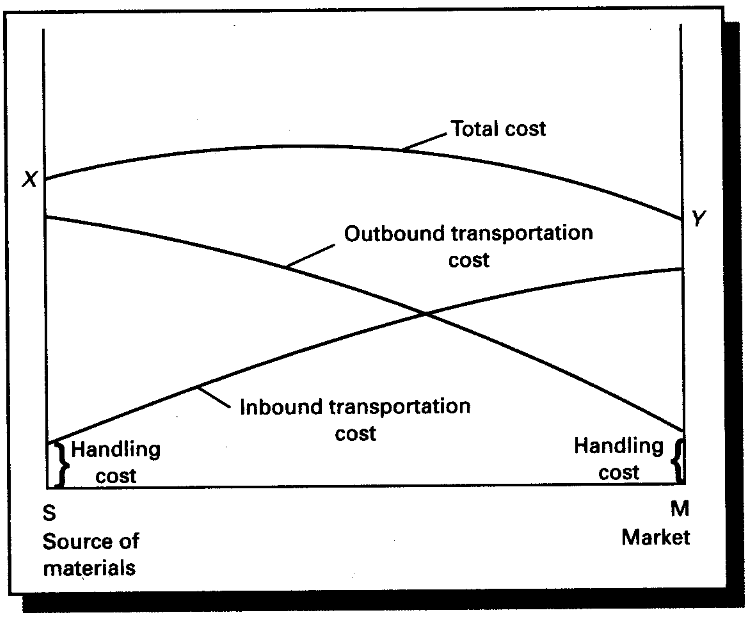
processes should be located as close to markets as possible (see Figure 13-2). An example of an industry that locates its plants in this manner is soft drink bottling. Syrups are shipped into the bottling plants and mixed with water. These plants are typically located in the general region of the markets for the products.

Finally, there are processes where there is no change in weight between raw materials and finished product. Assembly operations are representative of this category, where the finished product is a sum of the weight of the parts and components assembled into it. Such processes, according to Weber, are bound neither to the sources of the raw materials nor to the markets (see Figure 13-2). That is, the total of inbound and outbound transportation costs is the same at any location between source and market points.

Hoover's Tapered Transportation Rates

Hoover observed that transportation rates are tapered with distance. To minimize inbound plus outbound transportation costs where they are the dominant location force, a facility located between a raw material source and a market point will have a minimum transportation cost at one of these two points. As shown in Figure 13-3, location between these points is economically unstable. Since Y is lower than X on the cost curves, location should be at Y.

Figure 13-3
Tapered Transportation Rates Force Location to the Source of Materials or to the Market



SINGLE FACILITY LOCATION

Let us now turn to contemporary ways of looking at facility location. With the popularity of applied mathematics and computers, these approaches are mathematical in nature rather than conceptual. We begin with a popular model that is used for locating a single plant, terminal, warehouse, or retail or service point. It has been variously known as the exact center-of-gravity approach, *p*-median, the grid method, and the centroid method. The approach is simple, since the transportation rate and the point volume are the only location factors. This model is classified mathematically as a static continuous location model.

Where should the facility be located given a set of points representing source points and demand points, their volumes that are to be moved to or from a single facility of unknown location, and their associated transportation rates? We seek to minimize the sum of the volume at a point multiplied by the transportation rate to ship to the point multiplied by the distance to the point, which is the total transportation cost. That is,

$$\text{Min } TC = \sum_i V_i R_i d_i \tag{13-1}$$

where

TC = total transportation cost
V_i = volume at point i
R_i = transportation rate to point i
d_i = distance to point i from the facility to be located

The facility location is found by solving two equations for the coordinates of the location.[10] These exact center-of-gravity coordinates are

$$\overline{X} = \frac{\sum_i V_i R_i X_i / d_i}{\sum_i V_i R_i / d_i} \tag{13-2}$$

and

$$\overline{Y} = \frac{\sum_i V_i R_i Y_i / d_i}{\sum_i V_i R_i / d_i} \tag{13-3}$$

where

$\overline{X}, \overline{Y}$ = coordinate points of the located facility
X_i, Y_i = coordinate points of source and demand points

The distance d_i is estimated by

$$d_i = K\sqrt{(X_i - \overline{X})^2 + (Y_i - \overline{Y})^2} \tag{13-4}$$

where K represents a scaling factor to convert one unit of a coordinate point to a more common distance measure, such as miles or kilometers.

The solution process involves several steps, which are outlined as follows:

1. Determine the X,Y coordinate points for each source and demand point, along with point volumes and linear transportation rates.
2. Approximate the initial location from the center-of-gravity formulas by omitting the distance term d_i as follows:

$$\overline{X} = \frac{\sum_i V_i R_i X_i}{\sum_i V_i R_i} \tag{13-5}$$

and

$$\overline{Y} = \frac{\sum_i V_i R_i Y_i}{\sum_i V_i R_i} \tag{13-6}$$

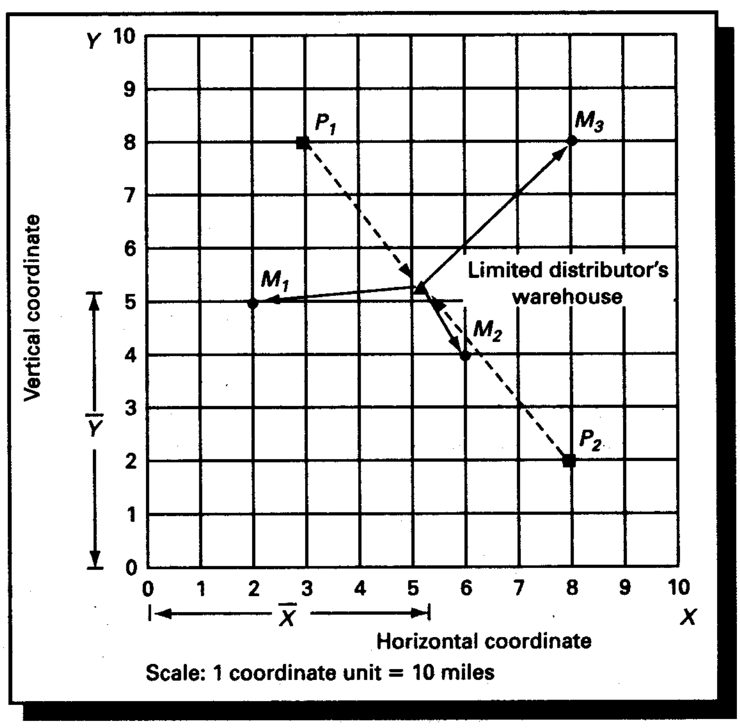
[10]These equations are derived from Equations (13-1) and (13-4) by taking the partial derivatives of TC with respect to X and Y, setting them equal to zero, and rearranging terms.

3. Using the solution for $\overline{X}, \overline{Y}$ from step 2, calculate d_i according to Equation (13-4). (The scaling factor K need not be used at this point.)
4. Substitute d_i into Equations (13-2) and (13-3), and solve for the revised $\overline{X}, \overline{Y}$ coordinates.
5. Recalculate d_i based on the revised $\overline{X}, \overline{Y}$ coordinates.
6. Repeat steps 4 and 5 until the $\overline{X}, \overline{Y}$ coordinates do not change for successive iterations, or they change so little that continuing the calculations is not fruitful.
7. Finally, calculate the total cost for the best location, if desired, by using Equation (13-1).

Example

Consider the problem of Limited Distributors, Inc., with two plants supplying the warehouse, which, in turn, supplies three demand centers. The spatial arrangement of the plants and market points is shown in Figure 13-4. We seek the location for the single warehouse that will minimize transportation costs. A grid overlay on a highway map is used as a convenience in establishing the relative point locations. Each plant and demand center location is expressed as a geometric coordinate point. Product A is supplied from P_1 and product B from P_2. These products are reshipped to the markets. Coördinate points, volumes, and transportation rates are summarized in Table 13-1.

Figure 13-4 Location Map of Plants P_1 and P_2 and Markets M_1, M_2, and M_3 and Suggested Warehouse Location



Point (i)	Product (s)	Total Volume Moving, V_i (cwt.)	Transportation Rate ($/cwt./mi.)[a]	Coordinates, X_i	Y_i
1-P_1	A	2,000	$0.050	3	8
2-P_2	B	3,000	0.050	8	2
3-M_1	A&B	2,500	0.075	2	5
4-M_2	A&B	1,000	0.075	6	4
5-M_3	A&B	1,500	0.075	8	8

[a]Determined by dividing a representative quoted rate ($/cwt.) by the distance (miles) over which the rate applies.

Table 13-1 Volume, Transportation Rates, and Coordinates for Market and Supply Points

Using Equations (13-5) and (13-6), we can find an initial, or approximate, location for the warehouse. Calculations are easy if we solve the equations in a tabular form. That is,

i	X_i	Y_i	V_i	R_i	V_iR_i	$V_iR_iX_i$	$V_iR_iY_i$
1	3	8	2,000	0.050	100.00	300.00	800.00
2	8	2	3,000	0.050	150.00	1200.00	300.00
3	2	5	2,500	0.075	187.50	375.00	937.50
4	6	4	1,000	0.075	75.00	450.00	300.00
5	8	8	1,500	0.075	112.50	900.00	900.00
					625.00	3,225.00	3,237.50

Now, we have

$$\overline{X} = 3{,}225.00/625.00 = 5.16$$

and

$$\overline{Y} = 3{,}237.50/625.00 = 5.18$$

These coordinates define the warehouse location, as shown in Figure 13-4. The total transportation cost associated with this location is determined in Table 13-2.

The previous example was terminated at step 2 of the solution process. This is an approximate solution. In many applications, it will provide a location that is reasonably close to the optimum. It will provide a first approximation to the least cost solution and will give an optimum when there is perfect symmetry in location, volume, and costs associated with the points. When these conditions are not fully met, research has shown that the potential error still can be quite small, if the volume associated with one or a few points is not substantially larger than the rest; there is a

Table 13-2
Calculation of Transportation Cost for Limited Distributors' Warehouse Location

i	X_i	Y_i	(4) V_i	(5) R_i	(6) d_i (MI.)[a]	(7) = (4)×(5)×(6) COST, $
1	3	8	2,000	0.050	35.52[b]	$ 3,552
2	8	2	3,000	0.050	42.64	6,395
3	2	5	2,500	0.075	31.65	5,935
4	6	4	1,000	0.075	14.48	1,086
5	8	8	1,500	0.075	40.02	4,503
	Total transportation cost					$21,471

[a] These distances have been rounded to the nearest 1/100 mile.

[b] From Equation (13-4), $d_i = 10\sqrt{(3-5.16)^2 + (8-5.18)^2} = 35.52$ mi.

large number of demand or supply points in the problem; and the transportation rates are linear, or nearly linear, with distance.[11] For example, a modest problem involving 50 demand points with randomly dispersed locations, volumes, and linear transportation rates had an average error of 1.6 percent from optimum when using this method. Of course, this error can increase substantially as the number of demand points is decreased.

Finding a more exact center-of-gravity solution requires completing the remaining steps in the solution process. We are not able to find the solution directly and must resort to an iterating procedure. A rather simple and straightforward method is that of successive approximations. Although there are others, this procedure serves us well in this application. It can be time-consuming for hand calculation, but lends itself nicely to computer solution.

Example

Continuing with the Limited Distributors' problem, we would now use the center-of-gravity solution as the starting solution in Equations (13-1) and (13-2) to find the exact location. The location coordinates for the first iteration can be found by solving the equations in the following tabular form, using results from the previous example.

i	(2) V_iR_i	(3) $V_iR_iX_i$	(4) $V_iR_iY_i$	(5) d_i	(6) = (2)/(5) V_iR_i/d_i	(7) = (3)/(5) $V_iR_iX_i/d_i$	(8) = (4)/(5) $V_iR_iY_i/d_i$
1	100.00	300.00	800.00	35.52	2.815	8.446	22.523
2	150.00	1,200.00	300.00	42.63	3.519	28.149	7.037
3	187.50	375.00	937.50	31.65	5.924	11.848	29.621
4	75.00	450.00	300.00	14.48	5.180	31.077	20.718
5	112.50	900.00	900.00	40.02	2.811	22.489	22.489
					20.249	102.009	102.388

[11] Ronald H. Ballou, "Potential Error in the Center of Gravity Approach to Facility Location," *Transportation Journal* (Winter 1973), pp. 44–49.

Table 13-3 One Hundred Computational Cycles of Location Coordinates and Total Transportation Costs As Generated from the COG Software Module

ITERATION	$\overline{X}$ COORD.	$\overline{Y}$ COORD.	TOTAL COST, $	
0	5.160	5.180	21,471.00	⇐ Center of gravity
1	5.038	5.057	21,431.22	
2	4.990	5.031	21,427.11	
3	4.966	5.032	21,426.14	
4	4.951	5.037	21,425.69	
5	4.940	5.042	21,425.44	
6	4.932	5.046	21,425.30	
7	4.927	5.049	21,425.23	
8	4.922	5.051	21,425.19	
9	4.919	5.053	21,425.16	
10	4.917	5.054	21,425.15	
11	4.915	5.055	21,425.14	
.	.	.	.	
.	.	.	.	
.	.	.	.	
100	4.910	5.058	21,425.14	⇐ Exact solution

The revised location coordinate points can be calculated as

$$\overline{X} = 102.009/20.249 = 5.038$$

and

$$\overline{Y} = 102.388/20.249 = 5.057$$

with a total cost of $21,431.

Using the computer software module in LOGWARE known as COG, we can complete 100 iterations of this procedure. The results are given in Table 13-3. In this problem, note that the total cost does not decline further after the eleventh iteration, and there is little change in the location coordinates. This is the nature of this particular problem, but other problems may show dramatic differences.

Extensions to the Single Facility Location Model

The continuous location nature and simplicity of the exact center-of-gravity approach, given its appeal either as a location model unto itself or as a submodel in more elaborate methods, has encouraged researchers to extend its capability. Primary among these extensions are to include customer service and revenues,[12] to handle multiple locations,[13] and to represent nonlinear transportation costs.[14]

[12]See Donald J. Bowersox, "An Analytical Approach to Warehouse Location," *Handling & Shipping*, vol. 2 (February 1962), pp. 17–20; and Ronald H. Ballou, *Business Logistics Management*, 2nd ed. (Upper Saddle River, NJ: Prentice Hall, 1985), pp. 311–314.
[13]See Allan E. Hall, "Program Finds New Sites in Multi-Facility Location Problem," *Industrial Engineering* (May 1988), pp. 71–74; and Ballou, *Business Logistics Management*, pp. 316–323.
[14]Leon Cooper, "An Extension of the Generalized Weber Problem," *Journal of Regional Science*, Vol. 8, No. 2 (1968), pp. 181–197.

Appraisal of Single Facility Location

In addition to the center-of-gravity model, other single facility location approaches include graphical techniques[15] and approximating methods.[16] All vary in the degree of realism that they portray, in their speed and ease of computation, and in their ability to guarantee an optimum solution. Clearly, no single model is likely to have all of the features desired for a particular location problem so that the solution will lead directly to a final decision and management can merely delegate location decisions to an analyst. Therefore, these models can only provide solution guidelines and their effective use requires a good understanding of their strengths as well as their shortcomings.

The benefit of these single location models is quite clear—they aid the search for the best solution to a location problem, and they capture enough of the reality of the actual problem so that the solution is meaningful to management. The shortcomings may not be so obvious, and they need to be noted. Although any model will exhibit some shortcomings when applied to a real problem, this does not mean that the model is not useful. What is important is the sensitivity of the location model's results to a poor representation of reality. If a simplifying assumption, such as linearity in transportation rates, has little or no effect on a model's suggestion for a facility location, a simpler model may well prove to be more effective than elegant ones.

Some of the simplifying assumptions in single location models are listed next.

1. Demand volumes are frequently assumed concentrated at one point, when in fact they are generated from a number of customer points that are dispersed over a wide area. The market center of gravity is often used as the demand cluster, but this is subject to some error in calculating transportation costs to the demand cluster instead of to individual demand points.
2. Single facility location models typically find a location based on variable costs. They make no distinction between the differences in capital cost required for establishing a warehouse at various locations and other costs such as labor, inventory-carrying costs, and utilities associated with operating a facility at different locations.
3. Total transportation costs usually are assumed to increase proportionately with distance; however, most transport rates are composed of a fixed component and a variable component that varies with distance. Rate minimums and rate blanketing may further distort their linearity.
4. Straight-line routes are commonly assumed between the facility and other network points. This is rarely true, since travel is over a defined road network, established rail system, or through a rectilinear city street network. A proportionality factor can be included in the model to convert straight-line distances to approximate highway miles, rail miles, or whatever. This conversion factor, called a *circuity factor*, varies by location. For U.S. intercity transport, calculated straight-line miles should be increased by 20 percent to get highway direct-route

[15]Alfred Weber, *Uber den Standort der Industrien.*

[16]G. O. Wesolowsky and R. F. Love, "A Nonlinear Approximation Method for Solving a Generalized Rectangular Distance Weber Problem," *Management Science*, Vol. 18(1972), pp. 656–663.

miles and by 24 percent to get rail short-line miles. For city streets, a factor of 41 to 44 percent can be used. A table of circuity factors for truck travel in different countries is given in Chapter 14.

5. There is some concern that location models such as these are not dynamic. That is, they do not find a solution that reflects future changes in revenues and costs.

Applications

- Leaseway Transportation Corporation was able to make use of the exact center-of-gravity model to locate a truck maintenance facility in Boston. The company leased varying numbers of trucks to many accounts throughout the Boston metropolitan area. The truck maintenance facility was to be located for maximum convenience of all accounts. The location of each account and the number of trucks leased was known. The transportation rate was the same throughout the region. The center-of-gravity model gave the general location within which a specific site could be selected.
- An oil company used the center-of-gravity method to locate oil collection platforms in the Gulf of Mexico. Many wellheads were located throughout the floor of the Gulf. A group of these was connected using pipe that moved the oil to a collection platform on the surface. The center-of-gravity method was appropriate for finding the collection platform location that would minimize the total length of pipe needed.

Multiple Facility Location

The more complex, yet more realistic, location problem for most firms occurs when two or more facilities must be located simultaneously, or additional facilities are to be located when at least one already exists. This problem is common because all but the smallest companies have more than one facility in their logistics systems. It is complex because these facilities cannot reasonably be treated as economically independent, and the number of possible location configurations becomes enormous.

Observation

A few years ago, a company producing industrial cleaning compounds sold its products in approximately 2,000 U.S. counties, used 105 public warehouses, and manufactured its products in four plants. There were more than 800,000 possible plant-warehouse-customer combinations to be considered among only the existing locations. Finding an optimum warehouse configuration was further complicated by the several hundred product items sold and several modes of transportation used.

Let us direct our attention to the warehouse location problem as a general class, since it is a common problem experienced by many kinds of businesses. It can be characterized by several basic planning questions:

1. How many warehouses should there be in the supply chain network? How large should they be, and where should they be located?
2. Which demand points should be assigned to a warehouse? Which warehouses should be assigned to each plant, vendor, or port?
3. Which products should be stocked in each warehouse? Which products should be shipped directly from plants, vendors, or ports to customers?

A number of location methods have been developed that aid in answering some or all of these questions. Several of these, although by no means an exhaustive selection, are presented here to show the variety and power of the approaches. Mathematical location methods may be categorized as exact, simulation, and heuristic.

Exact Methods

Exact methods refer to those procedures with the capability to guarantee either a mathematically optimum solution to the location problem, or at least a solution of known accuracy. In many respects, this is an ideal approach to the problem of location; however, the approach can result in long computer running times, huge memory requirements, and a compromised problem definition when applied to practical problems. Calculus[17] and mathematical programming models are examples of this approach, and both will be illustrated.

Multiple Center-of-Gravity Approach

The nature of the multiple facility location problem is seen if we use the exact center-of-gravity approach in a multilocation format. Recall that this is a calculus-based model that finds the minimum transportation cost solution for an intermediate facility located among origin and destination points. If more than one facility is to be located, then it is necessary to *assign* the origin and destination points to arbitrary locations. This forms clusters of points equal to the number of facilities being located. Then, an exact center-of-gravity location is found for each of the clusters. These assignments to the facilities can be made in many ways, especially when considering multiple facilities and a large number of origin and destination points in the problem. One approach is to form the clusters by grouping the points that are the closest to each other. After the center-of-gravity locations are found, the points are reassigned to these locations. New center-of-gravity locations are found for the revised clusters. The process is continued until there is no further change. This completes the computations for a specified number of facilities to be located. It can be repeated for different numbers of facilities.

As the number of facilities is increased, it is quite common for transportation costs to decline. In trade-off with these decreasing transportation costs are the

[17]For other calculus models, see Edward H. Bowman and John B. Stewart, "A Model for Scale of Operations," *Journal of Marketing*, Vol. 20 (January 1956), pp. 242–247; and Arthur M. Geoffrion, "Making Better Use of Optimization Capability in Distribution System Planning," *AIIE Transactions*, Vol. 11, No. 2 (June 1978), pp. 96–108.

increases in the total fixed costs and system inventory carrying costs. The best solution is the one that minimizes the sum of all of these costs.

Although this method is optimal if all the ways of assigning points to clusters are evaluated, it becomes computationally impractical for realistic-size problems. Assigning many customers to even a small number of facilities is an enormous combinatorial task. Another approach is needed.

Mixed Integer Linear Programming

Mathematicians have labored for many years to develop efficient solution procedures that have a broad enough problem description to be of practical value in dealing with the large, complex location problem frequently encountered in supply chain network design and yet provide a mathematically optimum solution. They have experimented with the use of sophisticated management science techniques, either to enrich the analysis or to provide improved methods for solving this difficult problem optimally. These methods are goal programming,[18] tree search methods,[19] and dynamic programming,[20] among others.[21] Perhaps the most promising of this class is the mixed integer linear programming approach.[22] It is the most popular methodology used in commercial location models.[23]

The primary benefit associated with the mixed integer linear programming approach—a benefit not always offered by other methods—is its ability to handle fixed costs in an optimal way. The advantages of linear programming in dealing with the allocations of demand throughout the network, which is at the heart of such an approach, are well known. Although optimization is quite appealing, it does exact its price. Unless special characteristics of a particular problem are exploited, computer-running times can be long and memory requirements substantial. There is no guarantee that the optimal solution will be found unless all possible alternatives are evaluated. Even if the optimal solution is found, slight changes in the data can cause subsequent runs to require substantial computational time.

A recent application of mixed integer linear programming is in the context of Reverse Logistics Network Design.[24]

[18]Sang M. Lee and Richard L. Luebbe, "The Multi-Criteria Warehouse Location Problem Revisited," *International Journal of Physical Distribution and Materials Management*, Vol. 17, No. 3 (1987), pp. 56–59.
[19]U. Akinc and B. M. Khumawala, "An Efficient Branch and Bound Algorithm for the Capacitated Warehouse Location Problem," *Management Science*, Vol. 23 (1977), pp. 585–594.
[20]Robert F. Love, "One-Dimensional Facility Location-Allocation Using Dynamic Programming" *Management Science*, Vol. 23, No. 6 (January 1976), pp. 614–617.
[21]Recall the survey of location methods by Brandeau and Chiu, "An Overview of Representative Problems in Location Research."
[22]A. M. Geoffrion and G. W. Graves, "Multicommodity Distribution System Design by Benders Decomposition," *Management Science*, Vol. 20, No. 5 (January 1974), pp. 822–844; P. Bender, W. Northrup, and J. Shapiro, "Practical Modeling for Resource Management," *Harvard Business Review*, Vol. 59, No. 2 (March-April 1981), pp. 163–173; and Jeffrey J. Karrenbauer and Glenn W. Graves, "Integrated Logistics Systems Design" in James M. Masters and Cynthia L. Coykendale, eds., "Logistics Education and Research: A Global Perspective," *Proceedings of the Eighteenth Annual Transportation and Logistics Educators Conference* (St. Louis, MO: October 22, 1989), pp. 142–171.
[23]Ronald H. Ballou and James M. Masters, "Commercial Software for Locating Warehouses and Other Facilities," *Journal of Business Logistics*, Vol. 14, No. 2 (1993), pp. 71–107.
[24]Samir K. Srivastava and Rajiv K. Srivastava,"Profit Driven Reverse Logistics," *International Journal of Business Research*, Vol. 4, No. 1 (2005), pp. 53–61.

Warehouse location problems are presented in many variations. Researchers who have applied the integer programming approach have described one such warehouse location problem as follows:

> There are several commodities produced at several plants with known production capacities. There is a known demand for each commodity at each of the number of customer zones. This demand is satisfied by shipping via warehouses, with each customer zone being assigned exclusively to a single warehouse. There are lower as well as upper limits on the allowable total annual throughput on each warehouse. The possible locations for the warehouses are given, but the particular sites to be used are to be selected so as to result in the least total distribution cost. The warehouse costs are expressed as fixed charges (imposed for the sites actually used) plus a linear variable charge. Transportation costs are taken as linear.
>
> Thus, the problem is to determine which warehouse locations to use, what size warehouse to have at each selected location, what customer zones should be served by each warehouse, and what the pattern of transportation flows there should be for all commodities. This is to be done so as to meet the given demands at minimum total distribution cost, subject to plant capacity and warehouse configuration of the distribution system.[25]

In descriptive language, this problem can be expressed in the following manner:

> Find the number, size, and locations of warehouses in a supply chain network that will minimize the fixed and linear variable costs of moving all products through the selected network subject to the following:
>
> 1. The available supply of the plants cannot be exceeded for each product.
> 2. The demand for all products must be met.
> 3. The throughput of each warehouse cannot exceed its capacity.
> 4. A minimum throughput of a warehouse must be achieved before it can be opened.
> 5. All products from the same customer must be served from the same warehouse.

The problem can be solved using general integer linear programming computer software packages such as GAMS.[26] Historically, such practical problems were not solved, even with the most powerful computers. Researchers now apply such techniques as decomposing a multiproduct problem into as many subproblems as there are products, eliminating parts of the problem irrelevant to the solution, and approximating data relationships in forms that complement the solution approach in order to achieve acceptable computer running times and memory requirements. Today, researchers are claiming to be able to extend substantially the number of echelons in the network that can be modeled, include multiple periods in the model, and cautiously handle nonlinear cost functions.[27]

[25]Geoffrion and Graves, "Multicommodity Distribution System Design," p. 822.
[26]Samir K. Srivastava and Rajiv K. Srivastava, "Profit Driven Reverse Logistics," *International Journal of Business Research*, Vol. 4, No. 1, 2005, pp. 53–61.
[27]Karrenbauer and Graves, "Integrated Logistics System Design."

Another location method that utilizes mixed integer programming is a modified *p-median* approach. It is less complicated and less robust than the previous formulation. Demand and supply points are located by means of coordinate points. Facilities are restricted to be among the demand or supply points. The costs affecting location are variable transportation rates expressed in units as \$/cwt./mi. and the annual fixed costs associated with the candidate facilities. The number of facilities to be located is specified before solution. The solution process finds this specified number among the candidate facilities.

Example

Environment Plus incinerates toxic chemicals used in various manufacturing processes. These chemicals are moved from 12 market areas around the country to its incinerators for disposal. The company provides the transportation due to the special equipment and handling procedures required. Transportation services are contracted at a cost of \$1.30 per mile and the trucks are fully loaded at 300 cwt. Trips are out and back from an incinerator. Therefore, the effective transport rate is \$1.30/mi. ×2 / 300 cwt. = \$0.0087/cwt./mi. The market locations, annual processing volume, and annual fixed operating costs, regardless of throughput volume, are shown in Table 13-4.

The metropolitan areas of Baltimore, Memphis, and Minneapolis will not permit the incinerators, and therefore are not candidate locations. If five locations are to be used, which should they be?

The PMED software module in LOGWARE can help to solve this problem. A database for this problem is available as PMED02.DAT. The results show the preferred locations to minimize the cost.

Table 13-4
Market Location, Volume, and Cost Data for Environment Plus

No.	Market	Annual Latitude,°	Fixed Longitude,°	Volume, cwt.	Operating Cost, \$
	Boston MA	42.36	71.06	30,000	3,100,000
	New York NY	40.72	74.00	50,000	3,700,000
2	Atlanta GA	33.81	84.63	170,000	1,400,000
	Baltimore MD	39.23	76.53	120,000	—
1	Cincinnati OH	39.14	84.51	100,000	1,700,000
	Memphis TN	35.11	89.96	90,000	—
	Chicago IL	41.84	87.64	240,000	2,900,000
	Minneapolis MN	44.93	93.20	140,000	—
3	Phoenix AZ	33.50	112.07	230,000	1,100,000
4	Denver CO	39.77	105.00	300,000	1,500,000
	Los Angeles CA	34.08	118.37	40,000	2,500,000
5	Seattle WA	47.53	122.32	20,000	1,250,000

No.	Facility Name	Volume	Assigned Node Numbers					
1	Cincinnati OH	680,000	1	2	4	5	7	8
2	Atlanta GA	260,000	3	6				
3	Phoenix AZ	270,000	9	11				
4	Denver CO	300,000	10					
5	Seattle WA	20,000	12					
	Total	1,530,000						
Total cost:	$9,455,339							

A map of the solution is shown in Figure 13-6.

Mixed integer linear programming has great appeal as a methodology, but the potentially long solution times of the method to handle large-scale location problems remains bothersome, although faster computers have helped. In addition, the difficulty of handling nonlinear functions as may occur in inventory policies, transportation rates, and sales and customer service relationships allows other approaches to remain competitive with mixed integer linear programming.

Simulation Methods

Although it may appear that location models providing mathematically optimum solutions are best, it should be remembered that the optimum solution to the

Figure 13-6 Plot of the Solution Results for the Environment Plus Location Problem

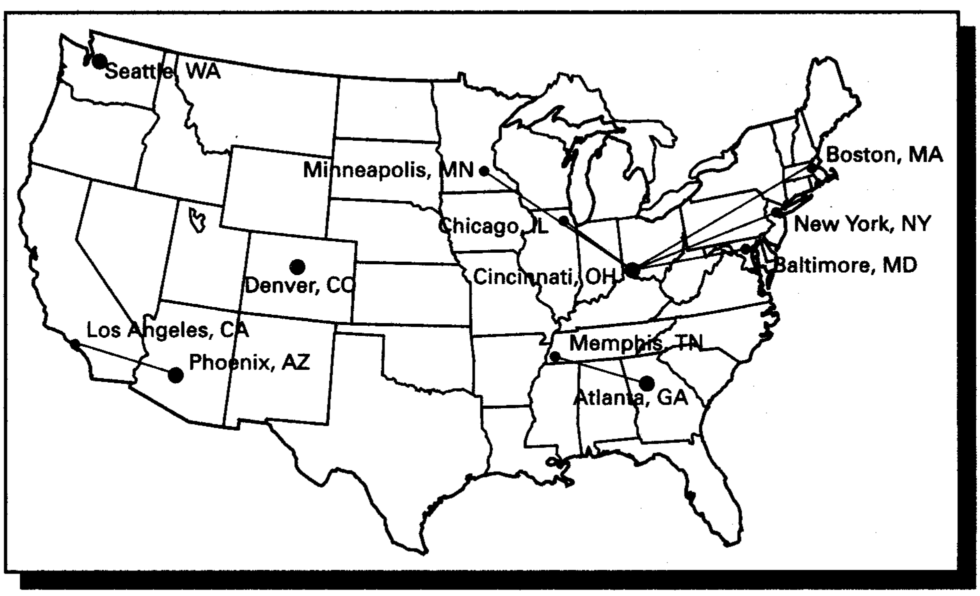
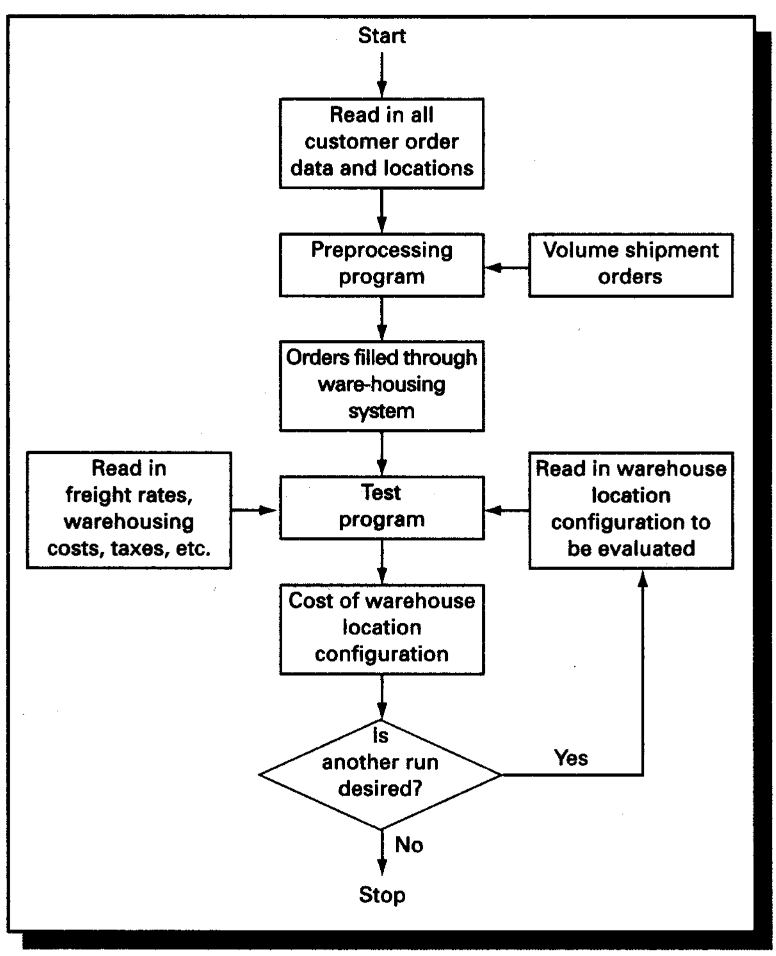


Figure 13-7
Flow Chart for a Warehouse Location Simulation Developed for the H. J. Heinz Company

Source: Harvey N. Shycon and Richard B. Maffei, "Simulation—Tool for Better Distribution," *Harvard Business Review*, Vol. 38 (November–December 1960), p. 73.



A desirable feature inherent in them is their ability to handle time-related aspects of inventory along with the geographical aspects of location. On the other hand, massive data requirements and long computer-running times can be a problem for this methodology. Nevertheless, the precise descriptions of reality are the primary reasons for their appeal.

A major problem with location simulators is that the user may not know how close the chosen warehouse configurations are to the optimum. Of course, we do know that the total cost curve for the location problem usually has a "flat bottom." Therefore, costs between closely ranked alternatives change little in the region of the optimum. As long as a reasonable number of prudently selected configurations have been evaluated, we can have a high degree of confidence that at least a satisfactory solution has been found.

Heuristic Methods

Heuristics can be referred to as any principles or concepts that contribute to reducing the average time to search for a solution. They are sometimes referred to as *rules of thumb* that guide problem solving. When applied to problems of location, such rules of thumb, which are a consequence of insight into the solution process, allow good solutions to be obtained quickly from numerous alternatives. Although heuristic methods do not guarantee that an optimum solution has been found, the benefits of reasonable computer running times and memory requirements, a good representation of reality, and a satisfactory solution quality are reasons to consider the heuristic approach to warehouse location.

Heuristic methods have been popular as a methodology for warehouse location. A classic yet still useful heuristic approach to the warehouse location problem was developed by Kuehn and Hamburger.[35] Other examples abound.[36] To help understand a heuristic model type for realistic problems, consider the nature of the location problem typically encountered in practice.

The location problem is one of trading off the costs relevant to location, which include

- Production and purchase costs
- Warehouse storage and handling costs
- Warehouse fixed costs
- Cost for carrying inventory
- Stock order and customer order processing costs
- Warehouse inbound and outbound transportation costs

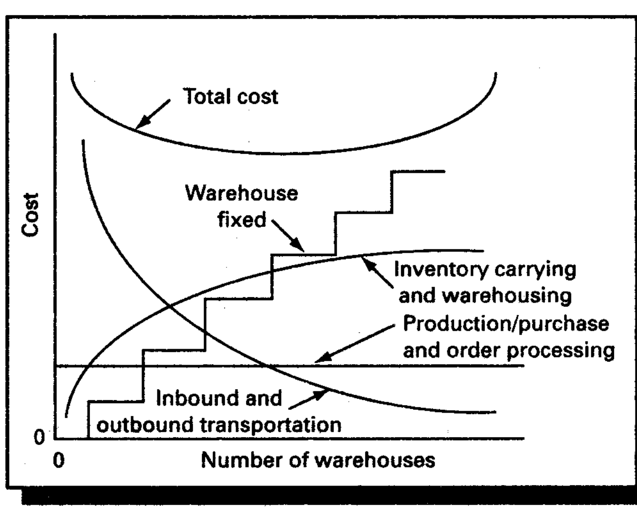
Each of these cost categories should reflect geographic differences, volume and shipping characteristics, policy variations, and economies of scale.

The nature of the cost trade-offs is graphically shown in Figure 13-8. Inventory, storage, and fixed costs are in direct trade-off with inbound and outbound transportation costs. Production and order processing costs also enter into the cost trade-off, but they cannot be adequately shown in this particular figure. The task of a location model is to seek out the warehouse/plant configuration that results in the minimum total relevant cost, subject to customer service and other practical restrictions placed on the problem.

Figure 13-8 shows that transportation costs decline with the number of warehouses in the distribution system. This is generally true because inbound shipments to a warehouse are made in larger quantities, and at lower rates, than outbound

[35]A. A. Kuehn and M. J. Hamburger, "A Heuristic Program for Locating Warehouses," *Management Science*, Vol. 10 (July 1963), pp. 643–666.
[36]Brandeau and Chiu, "An Overview of Representative Problems in Location Research," pp. 666–667; and Ronald H. Ballou and James M. Masters, "Commercial Software for Locating Warehouses and Other Facilities," *Journal of Business Logistics*, Vol. 14, No. 2 (1993).

Figure 13-8
Generalized Cost Trade-Offs in the Facility Location Problem



shipments. As more warehouses are put into the system, the warehouses are closer to customers, such that the inbound cost is increased but the outbound cost is disproportionately reduced. The transportation cost curve continues to decline in this manner until so many warehouses are used in the network that it is no longer practical to maintain full vehicle-load shipments to all warehouses. The transportation curve would turn up at this point.

The inventory-carrying and warehousing cost curve is shown to increase at a decreasing rate as the number of warehouses in the network increases. This is primarily a result of the inventory policy of a firm, as well as how that policy is executed, and the increasing amount of fixed cost in the network. With more warehouses, there is a proliferation of the amount of safety stock in the network. If the firm is controlling inventories by means of economical order quantity procedures, a tapered average inventory level and inventory-carrying cost curve results. Other policies may result in somewhat different inventory-carrying and warehousing cost curves, ranging from linear to tapered.[37] If the warehouses are privately owned or leased, there will be an annual fixed charge per warehouse. Total fixed costs in the network then will increase with the number of warehouses.

Selective Evaluation

A heuristic procedure can be developed from a method that has already been presented in this chapter, namely the multiple center-of-gravity method. The procedure is to solve for a specified number of facilities. Since the method only accounts for transportation costs, additional costs such as inventory and facility fixed costs may be added to create a more representative total cost. By repeating the procedure for various numbers of facilities, the best number of facilities, and their associated locations, can be found.

[37]Ronald H. Ballou, "Estimating and Auditing Aggregate Inventory Levels at Multiple Stocking Points," *Journal of Operations Management*, Vol. 1, No. 3 (February 1981), pp. 143–153.

Example

Suppose we have data for ten markets and their corresponding transportation rates, as given in file MCOG01.DAT of the LOGWARE software. The markets are shown in Figure 13-9. In addition, there is an annual fixed charge of $2,000,000 for each warehouse. All warehouses have enough capacity to handle the entire market demand. The amount of inventory in the logistics system is estimated from $I_T(\$) = \$6,000,000\sqrt{N}$, where N is the number of warehouses in the network. Inventory-carrying costs are 25 percent per year. Handling rates at the warehouses are all the same; therefore, they do not affect the location outcome. How many warehouses should there be, where should they be located, and which markets should be assigned to each warehouse?

Using the MULTICOG software module in LOGWARE and repeatedly solving for various numbers of warehouse, a spreadsheet can be developed as shown in Table 13-5.

Four warehouses yield the best cost balance. As seen in Figure 13-9, warehouses should be located in markets 3, 7, 9, and 10. Markets 2, 3, 4, and 5 are assigned to the warehouse at 3; markets 1, 6, and 7 are assigned to the warehouse at 7; markets 8 and 9 are assigned to the warehouse at 9; and market 10 is assigned to the warehouse at 10.

The selection evaluation approach is heuristic for several reasons. First, the multiple center-of-gravity method includes some rules that are used to determine initial

Figure 13-9 Markets for Example Problem and a Solution with Four Warehouses

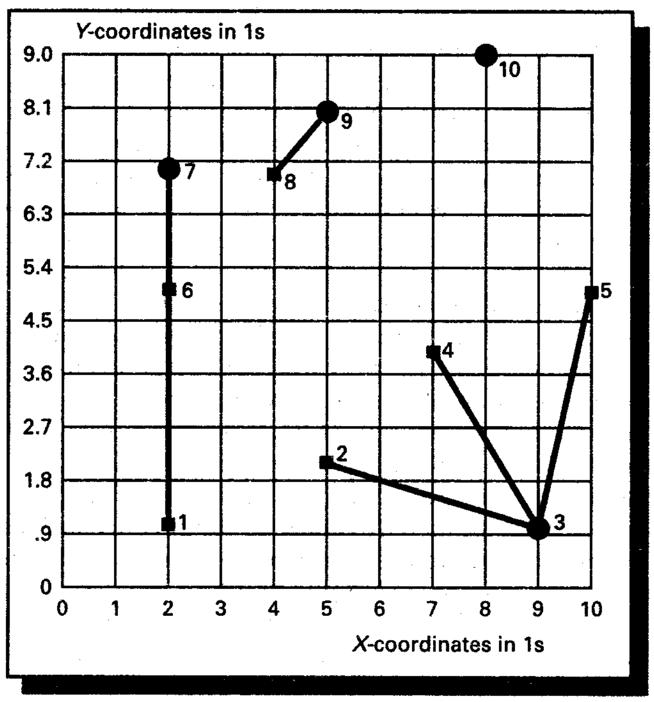


Table 13-5
Selection Evaluation Location Alternatives

Number of Warehouses	Transportation Cost, $	Fixed Cost, $	Inventory Cost, $	Total Cost, $	
1	41,409,628	2,000,000	1,500,000	44,909,628	
2	25,989,764	4,000,000	2,121,320	32,111,084	
3	16,586,090	6,000,000	2,598,076	25,184,166	
4	11,368,330	8,000,000	3,000,000	**22,368,330**	←
5	9,418,329	10,000,000	3,354,102	22,772,431	
6	8,032,399	12,000,000	3,674,235	23,706,634	
7	7,478,425	14,000,000	3,968,627	25,447,052	
8	2,260,661	16,000,000	4,242,641	22,503,302	
9	948,686	18,000,000	4,500,000	23,448,686	
10	0	20,000,000	4,743,416	24,743,416	

warehouse locations. This may cause sub-optimality in the results. Second, fixed costs and inventory costs are added to transportation costs *after* the warehouse locations are determined. It is preferred that these costs combine *during* the process that determines warehouse location for more optimal results. Regardless of its shortcomings, the approach has value when there is a minimum of information available to solve a location problem. It is useful in generating candidate locations that may be more thoroughly evaluated using procedures that are more robust.

Another form of selective evaluation specifies the number of warehouses to be evaluated and the particular warehouses in that number. Although the overall analysis is similar to that just presented using the center-of-gravity method, the analyst uses human judgment, logic, cognitive skills, and results from other model types to select the warehouses for evaluation. Since models seeking the optimum cannot hope to consider all the factors necessary for finding a satisfactory network design, this type of what-if analysis becomes very useful for practical network design. Linear programming is commonly used to allocate demand throughout the specified network. Choosing specific warehouses for evaluation is an effective way of dealing with practical issues in network design and making sure that desired warehouse combinations are considered. Most location analyses are dominated by selective evaluation of this type.

Selective evaluation may be used in solving location problems where the model used in the analysis is not primarily of a location nature. A common problem of this type is the location of a truck depot from which trucks are dispatched. Multiple trucks are routed to multiple stop-off points, and the route configuration is dependent on the location of the depot in proximity to the stop-off points. Depot location is dominated by transportation costs such that solving the truck routing problem is critical to depot location. A vehicle routing model such as ROUTER in LOGWARE can be used to form routes and minimize transportation cost. Then, selecting a particular depot location, solving the routing problem for the selected location, and adding costs specific to the location allow each location to be evaluated. This is a trial-and-error procedure, and a satisfactory solution to the location problem depends on the quality of the locations selected for evaluation.

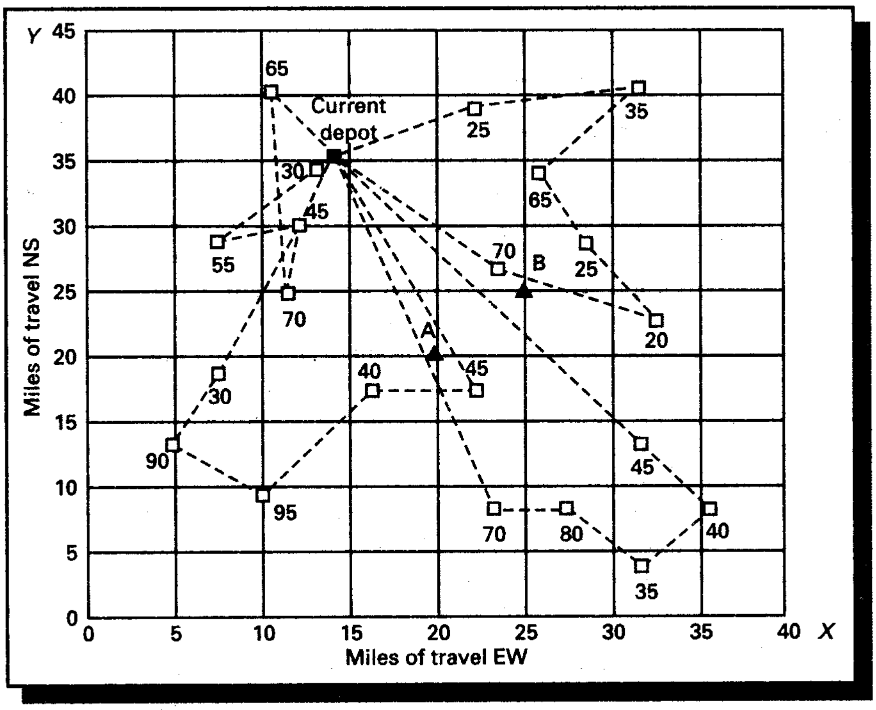


Figure 13-10 Current Depot Location with Stop Volumes in Cwt and Truck Routes

Example

A restaurant supply house makes deliveries to its customers (restaurants) on a daily basis. Currently, four trucks are dispatched on routes from a depot located in a city as shown in Figure 13-10 for a typical day's delivery volume. Considering that the trucks are amortized at a cost of $20 per day per truck, gas and maintenance costs to operate each truck are $0.40 per mile, and drivers are paid $11 per hour for wages and benefits, the current daily cost to serve the customers is $508. The company is considering a move to one of two central locations indicated in Figure 13-10 as *A* and *B*. The facility operating costs are expected to be about the same, but the amortized one-time moving cost is estimated as $40 per day. Using the ROUTER program in LOGWARE to generate routes from depots at *A* and *B* gives the revised transportation costs. Comparing the three alternatives on a daily cost basis, we have:

Location	Number of Trucks	Routing Cost	Truck Cost	Moving Cost	Total daily Cost
Current	4	$508	80	—	$588
A	5	497	100	40	637
B	4	484	80	40	604

Since the savings in routing costs cannot overcome the cost of moving to a new location, the economic decision is to keep the depot at the current location.

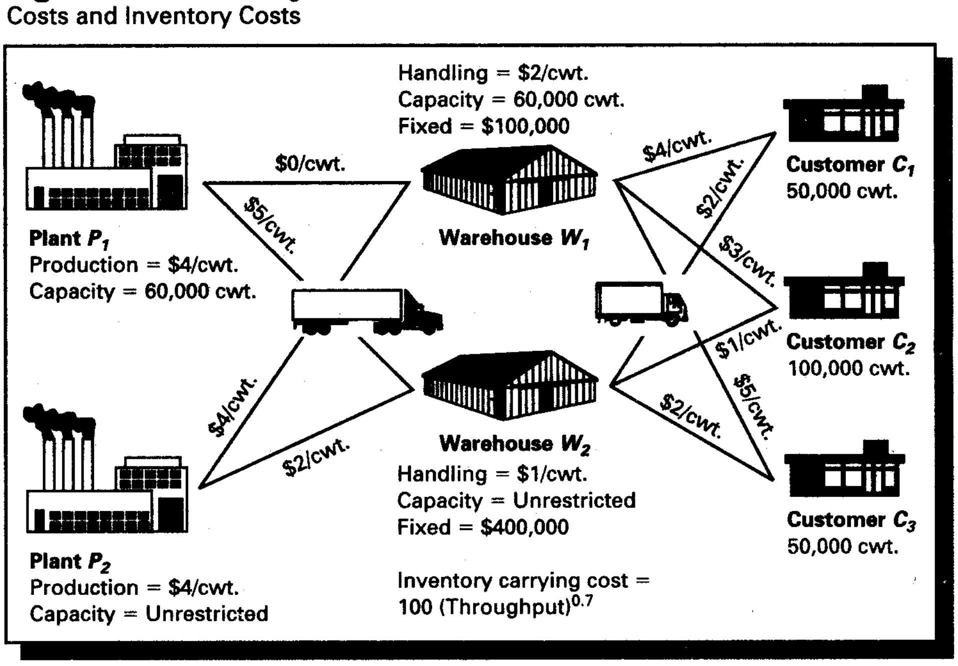
Guided Linear Programming

When serious heuristic procedures are developed for real-world location problems, they generally will include linear programming as part of the solution methodology. The appeal is that linear programming gives optimal results and can handle capacity restrictions that elude many other approaches. However, to be a truly robust procedure for location, fixed costs and nonlinear inventory costs need to be handled as well. Heuristic procedures need to be employed with linear programming to create an effective model.

Consider the small, single product problem shown in Figure 13-11. The first step is to construct a matrix that is formatted like the transportation problem of linear programming. By giving it a special structure, two logistics network echelons can be represented in the matrix of Figure 13-12. The heuristic process is guided by the manner in which the cell costs are entered into the matrix. Since the production and transportation costs between plants and warehouses are linear, they enter the plant-warehouse cells directly. For example, the cell cost representing the flow between P_2 and W_1 is the production plus transportation costs, or \$4/cwt. + \$4/cwt. = \$8/cwt.

The cellblock for warehouses and customers combines warehouse handling plus transportation plus inventory-carrying plus fixed costs. Handling and transportation rates can be read directly from Figure 13-11. However, there are no *rates* for inventory-carrying and fixed costs, and they must be developed, depending on each warehouse's

Figure 13-11 A Single-Product Location Problem with Warehouse Fixed Costs and Inventory Costs



		Warehouses		Customers			Plant and warehouse capacities
		W_1	W_2	C_1	C_2	C_3	
Plants	P_1	4[a] **60,000**	9	99[b]	99	99	60,000
	P_2	8	6 **140,000**	99	99	99	999,999[c]
Warehouses	W_1	0	99	9.7[d]	8.7 **60,000**	10.7	60,000
	W_2	99[b]	0	8.2[e] **50,000**	7.2 **40,000**	8.2 **50,000**	999,999[c]
Warehouse capacity and customer demand		60,000	999,999[c]	50,000	100,000	50,000	

[a] Production plus inbound transport rates, that is, 4 + 0 = 4.
[b] Used to represent an infinitely high cost.
[c] Used to represent unlimited capacity.
[d] Inventory carrying, warehousing, outbound transportation, and fixed rates, that is, 3.2 + 2 + 4 + 0.5 = 9.7.
[e] 3.2 + 1 + 2 + 2.0 = 8.2.

Figure 13-12 Matrix of Cell Costs and Solution Values for the First Iteration in the Example Problem

throughput. Since this throughput is not known, we must assume starting throughputs. For fixed costs, each warehouse is initially given the most favorable status by assuming that all demand flows through it. Thus, the rate associated with fixed costs for warehouse 1 would be the annual warehouse fixed cost divided by the total customer demand, or \$100,000/200,000 = \$0.50/cwt. For warehouse 2, it is \$400,000/200,000 = \$2.00/cwt.

For inventory-carrying costs, the per cwt. rate depends on the number of warehouses and the demand assigned to them. Again, to give each warehouse the greatest opportunity to be selected, the assumed throughput for the warehouses is equal, or the throughput for each warehouse is the total customer demand divided by the number of warehouses being evaluated. The "per unit" inventory-carrying cost is defined as the average inventory value in a warehouse divided by the warehouse period throughput, or $IC_i = K(\text{Throughput}_i)^a/\text{Throughput}_i$. Initially for each warehouse, the per cwt. inventory carrying cost is

Total customer demand

$$100[(200{,}000/2)^{0.7}]/(200{,}000/2) = \$3.2/\text{cwt.}$$

Number of warehouses

The estimated per unit fixed and inventory-carrying costs are now entered into the warehouse-customer cells of the matrix of Figure 13-12. The problem is solved in a normal manner using the TRANLP module of LOGWARE. The computational results are shown as the bold values in Figure 13-12. This now completes round one of the computations.

Subsequent computational rounds utilize warehouse throughputs from its previous round to improve upon the estimate of the per unit inventory-carrying and fixed costs for a warehouse. To make these estimates, we note from the solution that the throughput for W_1 is 60,000 cwt. and for W_2 it is 140,000 cwt. (see Figure 13-12). The allocated costs for the warehouses are

Warehouse	Per Unit Fixed Cost, \$/cwt.	Per Unit Inventory Carrying Cost, \$/cwt.
W_1	\$100,000/60,000 cwt. = 1.67	\$100(60,000 cwt.)$^{0.7}$/60,000 cwt. = 3.69
W_2	\$400,000/140,000 cwt. = 2.86	\$100(140,000 cwt.)$^{0.7}$/140,000 cwt. = 2.86

The cell costs in the matrix for warehouses to customers (see Figure 13-12) are recalculated to be:

	C_1	C_2	C_3
W_1	11.36[a]	10.36	12.36
W_2	8.72[b]	7.72	8.72

[a] 2 + 4 + 1.67 + 3.69 = 11.36
[b] 1 + 2 + 2.86 + 2.86 = 8.72

The remaining cells are unaltered. Now, solve the problem again.

The second iteration solution shows that all production is at plant 2 and all demand is to be served from warehouse 2. That is,

	C_1	C_2	C_3	
W_1	0	0	0	
W_2	50,000	100,000	50,000	⟵ Produced at plant 2

Subsequent iterations repeat the second iteration solution, since the allocation of inventory and fixed costs remain unchanged. A stopping point has been reached. To find the solution costs, recalculate them from the actual costs in the problem. Do not use the cell costs of Figure 13-12, since they contain the estimated values for warehouse fixed and inventory-carrying costs. Rather, compute costs as follows using the rates from Figure 13-11.

Cost Type	Warehouse 1 0 cwt.	Warehouse 2 200,000 cwt.	
Production	\$0	200,000 × 4	= \$800,000
Inbound transportation	0	200,000 × 2	= 400,000
Outbound transportation	0	50,000 × 2	= 100,000
		100,000 × 1	= 100,000
		50,000 × 2	= 100,000
Fixed	0		400,000

Inventory carrying	0	$100(200{,}000)^{0.7}$ = 513,714
Handling	0	200,000 × 1 = 200,000
Subtotal	$0	$2,613,714
Total	$2,613,714	

The previous example illustrates a heuristic procedure for a single product. However, many practical location problems require that multiple products be included in the computational procedure. With slight modification where fixed costs for a warehouse are shared among the products according to their warehouse throughput, the guided linear programming procedure can be extended to handle multiple product case.[38]

Appraisal of Multiple Facility Location Methods

Large-scale, multiple facility location models are impressive in the decision-making assistance that they provide a manager. Applications have ranged from large supply and distribution networks involving more than 100 warehouses, 20 product groups, 15 plants, and 300 customer demand zones to supply networks where hundreds of vendors supply a master warehouse that in turn supplies customers. Defense, retail, consumer goods, and industrial goods industries operating in both domestic and international environments have applied models of this scale. The primary reasons for location modelings' popularity are (1) they offer decision support to solving a problem of great consequence to management; (2) they are sufficiently robust to replicate a wide variety of logistics networks in acceptable detail for planning purposes; (3) they are inexpensive to apply, such that the benefits derived from their use far exceed their application cost; and (4) the data required by them are readily obtainable in most firms. These models have come a long way in representing reality since the early models of the land economists.

However, these models are not yet all they can be.[39] First, nonlinear and discontinuous cost relationships observed in inventory policies, transportation rate structures, and production and purchasing economies of scale continue to present mathematical difficulties in dealing with them accurately or efficiently. Second, facility location models should be expanded to deal more effectively with inventory and transportation decisions[40] simultaneously; that is, they should be truly integrated network planning models rather than requiring each problem to be dealt with in a separate, approximate way. Third, more attention should be given to incorporating revenue effects into the network design process, since the result generally is to rec-

[38]Ronald H. Ballou, "DISPLAN: A Multiproduct Plant/Warehouse Location Model with Nonlinear Inventory Costs," *Journal of Operations Management*, Vol. 5, No. 1 (November 1984), pp. 75–80.

[39]Ronald H. Ballou, "Unresolved Issues in Supply Chain Network Design" *Information Systems Frontiers*, Vol. 3, No. 4 (December 2001), pp. 417–425.

[40]For an example of transportation planning integration into location models, see Jossef Perl and Mark S. Daskin, "A Unified Warehouse Location-Routing Methodology," *Journal of Business Logistics*, Vol. 5, No. 1 (1984), pp. 92–111; and for inventory-holding costs integrated into location decisions, see Steven J. Erlebacker and Russell D. Meller, "The Interaction of Location and Inventory in Designing Distribution Systems," *IIE Transactions*, Vol. 32 (2000), pp. 155–166.

ommend more warehouses than when customer service is treated as a constraint and costs are minimized.[41] Fourth, the models should be made readily accessible to managers and planners so that they may be used frequently for tactical planning and budgeting rather than just for occasional strategic planning purposes. This will require closer ties to the firm's management information system so that the data to run them can be supplied immediately and in the form needed for model use.

Overall, each of these models, although they vary in terms of scope and solution procedures, can be used by the skilled analyst or manager to give valuable results. Making the existing technology easier to use and more accessible to the decision makers is a future direction in which development must move.

Dynamic Warehouse Location[42]

The location models discussed so far represent the type of sophisticated research that is being conducted to assist logisticians in solving practical warehouse location problems. Although many improvements have made the models more representative and computationally efficient, they remain essentially static in nature. That is, they do not provide optimal location patterns over time.

Demand and cost patterns shift over time, so implementing a location model's solution based on today's data may prove to be suboptimal under tomorrow's economic conditions. Optimal network configuration is a matter of changing from one configuration to another throughout a planning horizon to maintain an optimal configuration over time. This is not simply the problem of finding the best number, sizes, and locations of warehouses in each of the years throughout the planning horizon. There is a cost to change from one configuration to another. If the network uses public warehouses, it may be practical to change the configuration frequently, since there is little cost associated with closing out the stocks in one warehouse and starting them up in another. On the other hand, if there is a substantial cost of moving from one configuration to another, as might be the case if the warehouses are owned or leased, changing the network configuration should not occur often. Thus, it becomes important to implement the best design initially.

Finding the best configurations over time can be handled in several ways. First, the best warehouse locations can be found using the current conditions and those projected for some future year. The network configurations between the current year and the future year can then be averaged.

Second, the best current network configuration can be found and implemented. Then, in each year, as it arrives and data become available for the year, the best new

[41]Peng-Kuan Ho and Jossef Perl, "Warehouse Location under Service-Sensitive Demand," *Journal of Business Logistics*, Vol. 16, No. 1 (1995), pp. 133–162.
[42]This section is based on Ronald H. Ballou, "Dynamic Warehouse Location," *Journal of Marketing Research*, Vol. 5 (August 1968), pp. 271–276. Extensions to this work appear in D. Sweeney and R. L. Tatham, "An Improved Long-Run Model for Multiple Warehouse Location," *Management Science*, Vol. 22, No. 7 (March 1976), pp. 748–758; G. O. Wesolowsky and W. G. Truscott, "The Multi-Period Location-Allocation Problem with Relocation of Facilities," *Management Science*, Vol. 22 (1975), pp. 57–65; Tony Van Roy and Donald Erlenkotter, "A Dual-Based Procedure for Dynamic Facility Location," *Management Science*, Vol. 28, No. 10 (October 1982), pp. 1091–1105.

configuration is found. If the location savings between the new configuration and the previous one are greater than the costs associated with moving to the new configuration, the change should be considered. This method has the benefit of always working with actual data—not those that need to be forecasted.

Third, an optimal configuration path can be found over time that will precisely show when a change to a new configuration is needed and the configuration to which the change should be made. The methodologies that have already been discussed for static warehouse location can be incorporated into a dynamic programming procedure to find the optimal configuration path. A simple, single location problem can be used to illustrate the methodology.

Example

Suppose we have the problem as shown in Figure 13-13. A plant at Granville ships through a single warehouse to a number of markets at Arlington, Concordia, Stanton, Morton, and Chardon. It is projected that over time demand will increase and shift westward. The center-of-gravity locations for each year in the next five years are shown as points *A, B, C, D,* and *E.* The profits, discounted to the present,

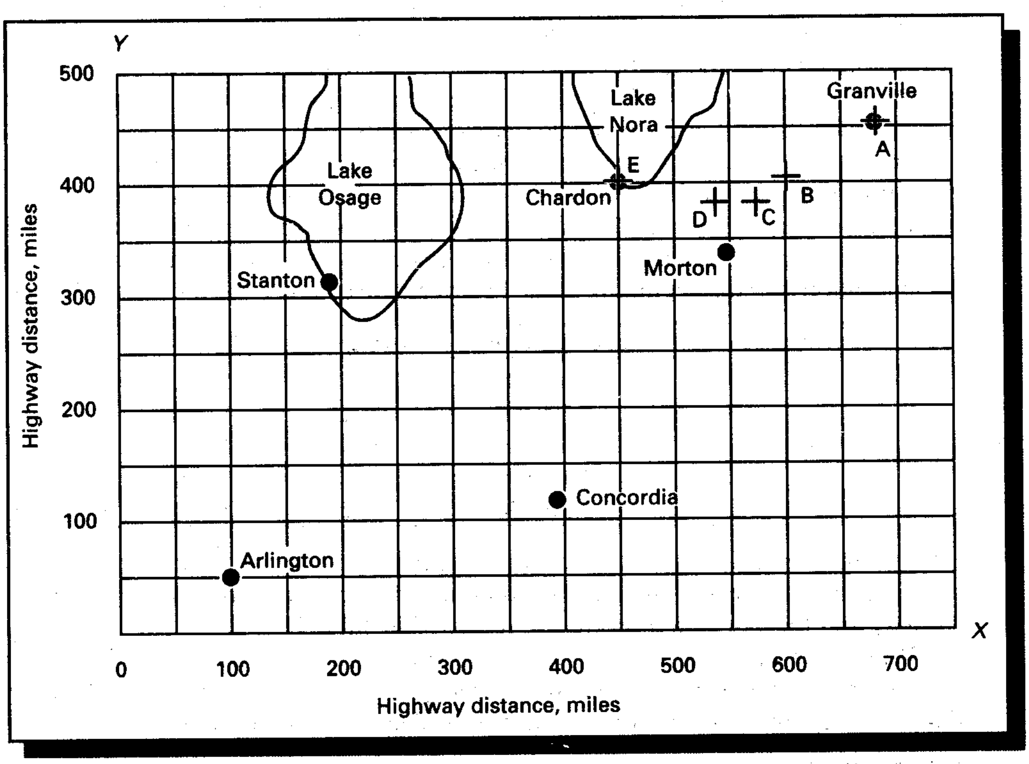


Figure 13-13 Plant-Market Location Map with Maximum-Profit Warehouse Location Points (+) for Each of Five Years

Warehouse Location Alternatives	Year from Present				
	1st	2nd	3rd	4th	5th
A	**$194,000**[a]	$ 356,100	$ 623,200	$ 671,100	$1,336,000
B	176,500	**372,000**[a]	743,400	750,000	1,398,200
C	172,300	344,700	**836,400**[a]	862,200	1,457,600
D	166,700	337,600	756,100	**973,300**[a]	1,486,600
E	159,400	303,400	715,500	892,800	**1,526,000**[a]

[a]These alternatives are the maximum profit locations for each year of the planning horizon, as shown in Figure 13-13.

Table 13-6 Projected Discounted Profits for Each Location in Each Year of the Planning Horizon with Maximum Profits Along the Main Diagonal

associated with each of these best locations, are given in Table 13-6. In addition, the discounted profits associated with locating in each of the other locations throughout the five years are also given. We know that it requires $100,000 to move from one location to another in any year. The cost of capital is 20 percent per year.

Finding the best location-relocation plan requires searching the profit table (Table 13-6) for the maximum profit path, after accounting for the appropriate moving charges. This is not an easy task, since, even for this small problem, there are $5^5 = 3125$ possible location-relocation plans. However, the technique of dynamic programming[43] can be applied here, and it will reduce the number of required computations to find the optimum plan to $5 \times 5 = 25$. Dynamic programming permits us to recast this multiperiod problem into a series of single-decision events.

Starting with the last year, we compute the profit associated with being in location A or of moving to one of the other locations. The discounted cost of moving in the beginning of year 5 would be $\$100{,}000/(1 + 0.20)^4 = \$48{,}225$. Given the location profits for the fifth year (see Table 13-6), we want to select the best strategy, assuming that we are in location A at the beginning of the fifth year. We evaluate the following choices:

	Alternative (x)	Location Profit		Moving Cost		Net Profit	
$P_5(A) =$	A	$1,336,000	−	0	=	$ 1,336,000	
	B	1,398,000	−	48,225	=	1,349,975	
	C	1,457,600	−	48,225	=	1,409,375	
	D	1,486,600	−	48,225	=	1,438,375	
	E	1,526,000	−	48,255	=	**1,477,775**	←

If the warehouse is located in A, we should move it to E to maximize profit.

We proceed to make similar computations for each location in the fifth year. The strategy and the associated profits are recorded in Table 13-7.

When strategy calculations are made for years other than year 5, we must include the profits accumulated from subsequent years. Consider the calculations that would be made for location D in year 3. The discounted moving cost would be

[43]For an introduction to dynamic programming, see Frederick S. Hillier, *Introduction to Operations Research*, 7th ed. (New York: McGraw-Hill, 2000), Chapter 10.

Table 13-7 Location-Relocation Strategies Over a Five-Year Planning Horizon with Cumulative Profits Shown from Year *j* to Year 5

Warehouse Location Alternatives (x)	Year from Present Date j: 1st $P_1(x)$	1st Strategy[a]	2nd $P_2(x)$	2nd Strategy[a]	3rd $P_3(x)$	3rd Strategy[a]	4th $P_4(x)$	4th Strategy[a]	5th $P_5(x)$	5th Strategy[a]
A	$3,719,086	S_A	$3,525,086	S_A	$3,168,986	M_C	$2,402,030	M_D	$1,477,775	M_E
B	3,717,486	S_B	3,540,986	S_B	3,168,986	M_C	2,402,030	M_D	1,477,775	M_E
C	[b] → **3,755,430**	S_C →	**3,583,130**	S_C →	**3,238,430**	S_C ↘	2,402,030	M_D	1,477,775	M_E
D	3,720,300	S_D	3,553,600	S_D	3,216,000	S_D	**2,459,900**	S_D →	**1,486,600**	S_D
E	3,659,197	S_E	3,499,797	M_C	3,168,986	M_C	2,418,800	S_E	1,526,000	S_E

[a] Strategy symbol refers to "staying" (*S*) in the designated location or "moving" (*M*) to a new location as indicated.
[b] Arrows indicate maximum profit location plan when warehouse is initially located at *C*.

$\$100,000/(1 + 0.20)^2 = \$69,444$. The location profits are found in Table 13-6. The cumulative profits for the subsequent year (year 4) are found in Table 13-7. We now can find the best strategy.

	Alternative (x)	Location Profit		Moving Cost		Cumulative Profit for Subsequent Years $P_4(x)$		Cumulative Profit for Year 3 $P_3(D)$
	A	\$623,200	−	69,444	+	\$2,402,030	=	\$2,955,786
	B	743,000	−	69,444	+	2,402,030	=	3,075,986
$P_3(D)$ = max.	C	836,400	−	69,444	+	2,402,030	=	3,168,986
	D	756,100	−	0	+	2,459,900	=	**3,216,000** ←
	E	715,500	−	69,444	+	2,418,800	=	3,064,856

Similar computations are carried out until Table 13-7 is complete. The optimum dynamic location can be traced through the table. We search the first year for the maximum cumulative profit (\$3,755,430), which is location C. From this point, the strategy indicated is $S_C\, S_C\, S_C\, S_D\, S_D$. This means to initially locate in C, stay in C for the first three years, and then switch to location D at the beginning of the fourth year. Stay in location D the remainder of the planning horizon. Note also in Table 13-7 that should we wish initially to locate in any of the other locations, we may trace an optimum strategy given that starting location.

Retail/Service Location

Retail and service centers are often the final stocking points in a physical distribution network. These include such facilities as department stores, supermarkets, branch banks, emergency medical centers, churches, recycling centers, and fire and police stations. Location analysis for these points often must be highly sensitive to revenue and accessibility factors rather than the cost factors so important to plant and warehouse location. Factors such as proximity to competition, population makeup, customer traffic patterns, nearness to complementary outlets, parking availability, proximity to good transportation routes, and community attitudes are just a few of the many factors that can influence retail or service location. Therefore, the previous methodology does not directly apply to these problems. Since the logistician is less likely to be directly responsible for retail or service location, we will examine only a few of the more popular methodologies.

Weighted Checklist

Often, many of the factors important to retail or service location are not quantified easily or inexpensively. Judgment remains an integral part of the location decision, yet it is difficult to make comparisons among sites unless the analysis can be quantified to some degree, even if crudely. One possibility is to form a weighted matrix of location factors, like that shown in Table 13-8, and score each factor for potential sites. An index number, which is the sum of the factor weights multiplied by the

Local Demographics	Site Characteristics
Population base of the local area	Number of parking spots available
Income potential in local area	Distance of parking areas to site
	Visibility of site from street
	Size and shape of the lot
	Condition of existing building (if any)
	Ingress and egress quality
Traffic Flow and Accessibility	**Legal and Cost Factors**
Number of vehicles	Type of zoning
Type of vehicles	Length of lease
Number of pedestrians	Local taxes
Type of pedestrians	Operations and maintenance
Availability of mass transit	Restrictive clauses in lease
Access to major highway	Voluntary regulations by local merchants
Level of street congestion	
Quality of access streets	
Retail Structure	
Number of competitors in area	
Number and types of stores in area	
Complementarity of neighboring stores	
Proximity to commercial areas	
Joint promotion by local merchants	

Source: Avijit Ghosh and Sara L. McLafferty, *Location Strategies for Retail and Service Firms* (Lexington, MA: D. C. Heath and Company, 1987), p. 49.

Table 13-8 An Example List of Factors Important to Retail or Service Site Selection

factor scores, is the total score for the site. Sites with high index values are preferable to sites with low index values.

Example

Suppose that a major paint manufacturer wants to site a retail outlet for its products. Outside experts, as well as standard checklists, would be consulted to generate a list of factors relevant to the problem of where to put the retail store. An abbreviated list of factors is shown in Table 13-9. Factor weights are assigned a number from 1 to 10, according to the relative importance of each factor, with 10 being most important. A particular site is scored on a scale from 1 to 10, with 10 representing the most favorable status. This particular site has a total index of 391. Other sites can be scored and the total index values compared. Of course, special care must be taken to score different sites consistently so that the index values can be reasonably compared.

Table 13-9
A Hypothetical Weighted Factor Checklist for a Retail Location Example

(1) Factor Weight (1 to 10)[a]	Location Factors	(2) Factor Score (1 to 10)[b]	(3) = (1) × (2) Weighted Score
8	Proximity to competing stores	5	40
5	Space rental/lease considerations	3	15
8	Parking space	10	80
7	Proximity to complementary stores	8	56
6	Modernity of store space	9	54
9	Customer accessibility	8	72
3	Local taxes	2	6
3	Community service	4	12
8	Proximity to major transportation arteries	7	56
	Total index		391

[a]Weights approaching 10 indicate great importance.
[b]Scores approaching 10 refer to a favored location status.

Observations

- When Dave Thomas, founder of Wendy's, was asked how his firm decided on new locations for its restaurants, he replied, "We see where a McDonald's restaurant is and locate as near to it as possible."
- The Original Mattress Factory, which was founded by a former CEO of the Sealy Mattress Company (the largest mattress producer in the United States), established a factory and retail store and promoted its mattresses heavily through advertising on radio and television. It was not long before competing mattress retail stores were located immediately next door or directly across the street.

Spatial-Interaction Model

One of the most popular approaches to determining the drawing power, or overall desirability, of a site is the gravity model. An early version was known as Reilly's law of retail gravitation,[44] which is remarkably similar to Newton's law of gravity. The basic idea is that two competing cities attract trade from an intervening town in direct proportion to each city's population but in inverse proportion to the square of distances between the cities and town. Although this model is quite simplistic, it has been enriched through using the *mass* or *variety* offered by a retail out-

[44]William J. Reilly, *The Law of Retail Gravitation* (New York: Knickerbocker Press, 1931).

let instead of *population*. Mass variables are square footage of the store, number of different items in stock, levels of inventory maintained, or other features that can attract customers. *Distance* in the original formula becomes customer driving distance or driving time to competing retail outlets and the proposed site. The power to which distance or driving time is raised can be empirically determined, usually by scaling from a map or driving the actual routes, to better reflect how distance or time repels trade.

The gravity concept has been modified into a more practical working model by Huff.[45] This spatial-interaction model developed an empirical basis for how consumers trade attractiveness of alternate retail sites with accessibility. This model is expressed as

$$E_{ij} = P_{ij}C_i = \frac{S_j/T_{ij}^a}{\sum_i S_j/T_{ij}^a} C_i \qquad (13\text{-}7)$$

where

- E_{ij} = expected demand from population center i that will be attracted to retail location j
- P_{ij} = probability of customers from population center i traveling to retail location j
- C_i = customer demand at population center i
- S_j = size of the retail location j
- T_{ij} = travel time between population center i and retail location j
- n = number of retail locations j
- a = empirically estimated parameter[46]

Note that size S may include all variables that attract customers to a retail site (store attractiveness, inventory availability, price, parking space, etc.). The retail site may be a single outlet or a service center of a group of outlets such as a shopping center. Travel time T may include all variables that repel customers (distance, traffic congestion, limitations to access, detours, etc.). The purpose of the model is to estimate the share of the total market that will be captured by various retail and service center sites.

The Huff model is a basic model for spatial interaction. Over the years, researchers have improved its ability to represent real problems, have reformulated it as a multiplicative model, and have suggested different definitions of the variables to improve its predictive performance.[47]

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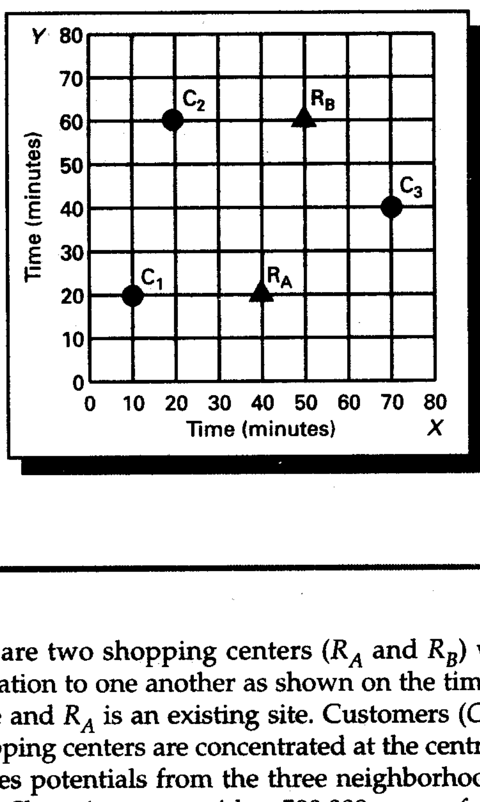
[45]David L. Huff, "A Computer Program for Location Analysis," in Raymond M. Hass (ed.), *Science, Technology, and Marketing*, (Chicago: American Marketing Association, 1966), pp. 371–379.
[46]One way to determine this parameter is to compare actual sales for an existing retail configuration with the model-generated sales. The parameter is set so that the two are equal. For another way to calibrate the model, see Avijit Ghosh and Sara L. McLafferty, *Location Strategies for Retail and Ser ice Firms* (Lexington, MA: Heath, 1987), pp. 95–100.
[47]For a review of these extensions, see Ghosh and McLafferty, Chapter 5.

Figure 13-14
A Time-Grid Map for Shopping Center Location Example



Example

Suppose that there are two shopping centers (R_A and R_B) within a metropolitan region located in relation to one another as shown on the time map in Figure 13-14. R_B is a potential site and R_A is an existing site. Customers (C_1, C_2, and C_3) who are attracted to the shopping centers are concentrated at the centroids of their neighborhoods. The total sales potentials from the three neighborhoods are \$10, \$5, and \$7 million, respectively. Shopping center *A* has 500,000 square feet of selling area and *B* has 1 million square feet. The parameter *a* is estimated to be 2.

The market share for each center can be approximated as shown in Table 13-10. First, the travel time is computed by using the coordinate point locations. For example, the travel time between C_1 and R_B is $D_{1B} = \sqrt{(X_1 - X_B)^2 + (Y_1 - Y_B)^2} = \sqrt{(10 - 50)^2 + (20 - 60)^2} = 56.6$. Second, the repelling time T^a is found—$T^2_{1B} = 3,200$.

Table 13-10 Estimated Total Shopping Center Sales for Example Problem

	Time from Customer *i* to Location *j*		T^2_{ij}		S_j/T^2_{ij}		$P_{ij} = \frac{S_j / T^2_{ij}}{\sum_j S_j / T^2_{ij}}$		$E_{ij} = P_{ij}C_i$	
Customer *i*	A	B	A	B	A	B	A	B	A	B
C_1	30.0	56.6	900	3200	555	313	0.64	0.36	\$6.4	\$3.6
C_2	44.7	30.0	2000	900	250	1111	0.18	0.82	0.9	4.1
C_3	36.1	28.3	1300	800	385	1250	0.24	0.76	1.7	5.3
					Total shopping center sales (\$ million)				\$9.0	\$13.0

Third, the probability P_{ij} of a dollar of revenue flowing to a center is computed. For example, the probability of customer 1 selecting center B is

$$P_{1B} = \frac{1{,}000{,}000/3{,}200}{(500{,}000/900) + (1{,}000{,}000/3{,}200)} = \frac{312.5}{868} = 0.36$$

Fourth, the probability multiplied by the total sales potential of a customer neighborhood is the sales contribution that each neighborhood makes to the shopping center sales. The expected contribution of neighborhood C_1 to R_B would be $0.36 \times 10 = \$3.6$ million. Finally, the neighborhood contributions are summed to give total shopping center sales. In this case, the potential site R_B should be able to generate $13 million in sales. This expected revenue can now be compared to operating costs, rents, and construction costs to see if the investment should be made.

Other Methods

A variety of additional methods plays a role in solving retail or service location problems. Regression analysis is important to forecasting the revenues that a specific site can expect. Covering models[48] are particularly useful for locating emergency services such as police and fire stations. Game theory has been suggested where competition is a key factor.[49] Location-allocation models such as goal programming and integer programming can be used. Consider an example of the use of integer programming to locate a bank's principal place of business.

Example[50]

The Ohio Trust Company wishes to locate within the 20 counties of northeast Ohio where it does not now have a principal place of business. According to the banking laws in Ohio, if a banking firm establishes a principal place of business (PPB) in any county, then branch banks can be established in that county and in any adjacent county. Ohio Trust would like to know in which counties to establish a minimum number of PPBs.

The 20 counties of northeastern Ohio are identified in Figure 13-15. In Table 13-11, those counties that are adjacent to each of the counties are listed.

To solve this problem as an integer-programming problem, we define

$$x_i = 1 \text{ if a PPB is to be located in county } i\text{; 0 otherwise}$$

[48]C. S. Craig and A. Ghosh, "Covering Approaches to Retail Facility Location" in *AMA Educators Proceedings* (Chicago: American Marketing Association, 1984).

[49]K. S. Moorthy, "Using Game Theory to Model Competition," *Journal of Marketing*, Vol. 22 (1985), pp. 262–282.

[50]Based on a problem shown in David R. Andersen, Dennis Sweeney, and Thomas A. Williams, *An Introduction to Management Science*, 5th ed. (St. Paul: West Publishing Co., 1988), pp. 335–339.

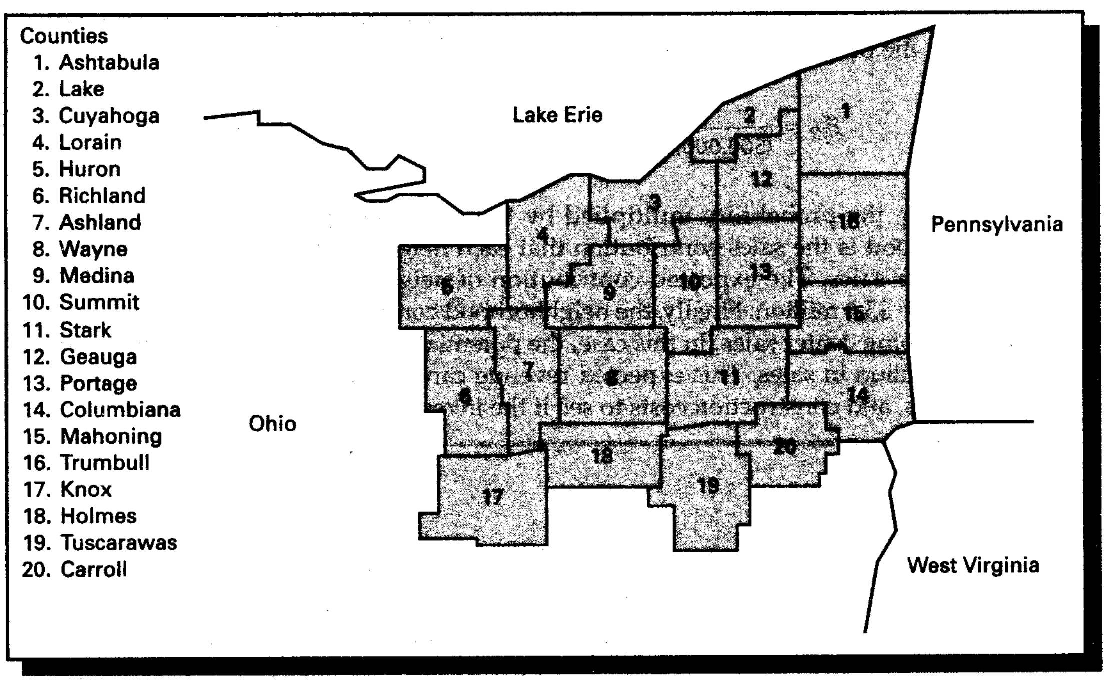


Figure 13-15 Northeastern Ohio Counties for Possible Principal Business Location by Ohio Trust Company

Source: Reprinted by permission from David R. Andersen, Dennis Sweeney, and Thomas A. Williams, *An Introduction to Management Science*, 5th ed. (St. Paul: West Po., 1988), p. 336.

Table 13-11 Adjacent Counties to Each County Under Consideration for Ohio Trust Company

Counties Under Consideration	Adjacent Counties by Number	Counties Under Consideration	Adjacent Counties by Number
1. Ashtabula	2,12,16	11. Stark	8,10,13,14,15,18,19,20
2. Lake	1,3,12	12. Geauga	1,2,3,10,13,16
3. Cuyahoga	2,4,9,10,12,13	13. Portage	3,10,11,12,15,16
4. Lorain	3,5,7,9	14. Columbiana	11,15,20
5. Huron	4,6,7	15. Mahoning	11,13,14,16
6. Richland	5,7,17	16. Trumbull	1,12,13,15
7. Ashland	4,5,6,8,9,17,18	17. Knox	6,7,18
8. Wayne	7,9,10,11,18	18. Holmes	7,8,11,17,19
9. Medina	3,4,7,8,10	19. Tuscarawas	11,18,20
10. Summit	3,8,9,11,12,13	20. Carroll	11,14,19

Source: Reprinted by permission from David R. Andersen, Dennis Sweeney, and Thomas A. Williams, *An Introduction to Management Science*, 5th ed. (St. Paul: West Po., 1988), p. 337.

Based on the data in Table 13-11, we can formulate the problem as follows:

Min. $1x_1 + 1x_2 \cdots + 1x_{20}$ ← County

subject to

$1x_1 + 1x_2 + \quad +1x_{12} + 1x_{16} \geq 1$ Ashtabula

$1x_1 + 1x_2 + 1x_3 + 1x_{12} \geq 1$ Lake

1 if Lake County or adjacent to Lake County

⋮ ⋮

$1x_{11} + 1x_{14} + 1x_{19} + 1x_{20} \geq 1$ Carroll

A constraint for each county

All xs = 0 or 1 ← Variables are integer

Using any appropriate integer programming code to solve this problem (e.g., MIPROG in LOGWARE), we find that 3 PPBs are needed, and they should be located in Ashland, Stark, and Geauga counties.

Service facilities may also be located by the integer programming method.

Example

MetroHealth Hospital would like to position emergency services in the surrounding communities of the large metropolitan area in which it operates. The objective is to have no patient drive more than ten minutes to reach an emergency room. The estimated patient driving times in minutes to potential locations is given as follows:

	To Potential Emergency Room Site					
From Neighborhood	1	2	3	4	5	6
A	0	5	15	25	25	15
B	5	0	20	30	15	5
C	15	20	0	10	25	15
D	25	30	i10	0	10	20
E	25	15	25	10	0	9
F	15	5	15	20	9	0

What is the minimum number of emergency rooms and where should they be located?

To solve this problem, we first note which emergency room sites are within the required ten-minute drive. The list is

Potential Site	Neighborhood
1	A, B
2	A, B, F
3	C, D

4	C, D, E
5	D, E, F
6	B, E, F

We can now write

$$\text{Minimize } X_A + X_B + X_C + X_D + X_E + X_F$$

subject to

$$\begin{aligned}
X_A + X_B \qquad\qquad\qquad\qquad\qquad & \geq 1 \text{ (Site 1 constraint)} \\
X_A + X_B \qquad\qquad\qquad\qquad + X_F & \geq 1 \text{ (Site 2 constraint)} \\
X_C + X_D \qquad\qquad\qquad & \geq 1 \text{ (Site 3 constraint)} \\
X_C + X_D + X_E \qquad\qquad & \geq 1 \text{ (Site 4 constraint)} \\
X_D + X_E + X_F & \geq 1 \text{ (Site 5 constraint)} \\
X_B \qquad\qquad\qquad + X_E + X_F & \geq 1 \text{ (Site 6 constraint)}
\end{aligned}$$

All Xs = 0 or 1

Solving this problem using the MIPROG module in LOGWARE gives $X_B = 1$ and $X_D = 1$, meaning that emergency rooms should be placed in neighborhoods *B* and *D*.

Other Location Problems

There are so many location problems occurring in supply chain planning that it is not practical to thoroughly discuss all of them. However, following are a few additional problem that may use the solution methodology already presented or that may require specialized solution procedures. They are selective illustrations of the variety of problem types faced by logisticians.

Hub and Spoke

A location problem solution made popular by the airlines, small package delivery services (FedEx and UPS), and communication systems is the hub-and-spoke concept. Rather than moving traffic directly from origin to destination, the traffic is directed through one or two hubs, or transfer facilities. Traffic moves from a hub to destination points or through a high-volume interconnection to another hub. The design problem is to minimize the transportation cost plus the cost of hub operation by (1) determining the number of hubs, (2) specifying their locations, and (3) routing traffic through the hubs. Since the origin and destination identities must be tied together, the problem is not solved in the same manner as a warehouse location problem. Rather, to solve the problem precisely requires a specialized algorithm.[51]

The model is highly suitable in the contexts where the present volumes are low, but the future market potential is high. This is especially true for rural and semiurban markets in the Indian context, and most companies presently are adopting this model.

[51]Hasan Pirkul and David A. Schilling, "An Efficient Procedure for Designing Single Allocation Hub and Spoke Systems," *Management Science*, Vol. 44, No. 12 (December 1998), pp. S235–S242.

Retail chains in India are spreading toward smaller towns and cities because of saturation in the retail market in metro cities. Retailers including Big Bazaar, Westside, and FoodWorld are adopting the hub-and-spoke approach and appointing franchises for reducing property costs by 20–25 percent. They build hubs of 50,000–60,000 square feet in big cities supported by warehouse facilities. These centers supply goods to small shops of 5,000–10,000 square feet in smaller localities within a distance of 100–150 kilometers.[52] Similarly, Godrej Agrovet is shifting to hub-and-spoke model from being a stand-alone outlet. It will open 1,000 hub-and-spoke centers in rural and semiurban areas in the next five years. The outlets will sell agro-products including seeds, pesticides, fertilizers, and grocery, apparel, footwear, home appliances, furniture and kitchen appliances. The centers will offer technical services such as farm management and soil micronutrient analyses to farmers. The hub will cover 10,000 square feet at a cost of Rs. 75 lakh, while spoke will cover 3,000 square feet at a cost of Rs. 30 lakh. The company expects revenues of Rs. 3,500–4,000 crore from the stores in the next five years.[53] This has led to demand of smaller transportation vehicles. Tata Motors' ACE mini-truck is being positioned as last mile distribution vehicle to complete the hub-and-spoke model.[54]

Obnoxious Facilities

It is common that location is judged on cost minimizing or profit maximizing criteria. This has the tendency to place locations close to the centers of demand, which is sometimes a disadvantage. Obnoxious facilities such as waste dumps, water treatment plants, chemical reclamation plants, and prisons are located on a criterion that attempts to maximize the minimum distance between them and the population. The problem can be solved in a manner similar to the MetroHealth Hospital emergency room problem discussed above. The difference is to develop a matrix of population centers that exceed the minimum required distance from a potential location. The problem constraints are then found from this matrix. Integer programming is used to solve the problem.

Microlocation

Practical location problems often involve significant-size geographical areas where distance approximations over road, rail, water, and air networks are reasonable. However, where location involves small demand areas such as locating newspaper distribution terminals, truck delivery and pickup terminals, workstation location in plants, and product location in warehouses, inaccuracies in estimating travel distances cannot be tolerated. Although the methodology for these microlocation problems is not necessarily different from those previously described, the requirement for precise data can be.

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[52] *Realty Plus* (November 30, 2004), p. 34.
[53] *Daily News & Analysis (DNA)* (July 6, 2006), p. 26.
[54] *The Hindu* (May 6, 2005).

CONCLUDING COMMENTS

Locating facilities in the network can be considered the most important logistics/SC strategic planning problem for most firms. It sets the conditions for the proper selection and good management of transport services and inventory levels. In many ways, facility location is the "bones" of the supply chain. Because there are often many facilities to be considered at one time, along with multiple products placed in them, multiple sources to serve them, and multiple customers served by them, the problem is often quite complex. Decision aids are typically useful.

The purpose of this chapter was to survey some of the more practical methods for the location of plants, warehouses, and retail or service facilities in the logistics network. We began with classifying location problems into a limited number of categories so that the major characteristics of location methodology could be identified. Next, the history of location theory was highlighted.

Single facility location methodology was represented by the exact center-of-gravity approach. This continuous location method is useful where transportation costs are the dominant location factor and there is to be no selection of candidate locations for testing, as in the case with mathematical programming methods.

Multiple facility location is the more important problem to most businesses. Three location approaches are typically used: (1) optimization; (2) simulation; and (3) heuristic methods. Although there are many models of each class that have been formulated, only one or two in each class have been used to illustrate the nature of the methodology. The extensions of both the static single and multiple facility models to deal with the problem of location over time were shown.

Finally, the problem of retail and service location was discussed. Several models were shown (weighted checklist and gravity models). The retail or service location problem stands in sharp contrast with the warehouse location problem primarily because it is revenue-based rather than cost-based, as is most warehouse location.

QUESTIONS

1. Refer to Weber's classification of industries. Should the following processes be (1) located near their markets; (2) located near their raw material sources; or (3) not necessarily located at markets or raw material sources?
   a. Bottling windshield washer fluid
   b. Assembling VCRs
   c. Smelting aluminum
   d. Refining crude oil
   e. Making apple cider

   How do transportation costs influence your suggestion?
2. According to Hoover, what characteristic in transportation rates makes location inherently unstable between markets and raw material sources?
3. What is a ubiquity? What impact does it have on location?
4. Multiple facility location models can be classified as exact, simulation, or heuristic. Explain the differences between these and cite examples of each type. Be sure to indicate why your example illustrates the type.

5. Why are single location methods not very appropriate for the multiple location problem?
6. What are the relevant costs for a multiple warehouse location problem? Why are these important to a proper location analysis?
7. What benefit does a "flat bottom" total cost curve have in making simulation a usable methodology for multiple warehouse location?
8. What is a heuristic method? What is an exact method? How are they useful in solving warehouse location problems?
9. What is dynamic warehouse location? When is it most appropriate to use this approach?
10. When is a weighted checklist a useful methodology for location?
11. In the location of a McDonald's restaurant, what factors would attract customers to particular locations? What factors would repel them? How would you go about determining the relative importance of each factor?

PROBLEMS

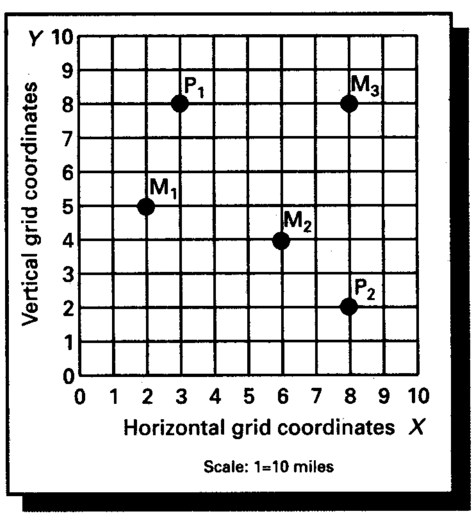
Some problems in this chapter can be solved or partially solved with the aid of computer software. The software packages in LOGWARE that are most important for this chapter are COG (C), MULTICOG (*M*), TRANLP (*T*), ROUTER (*R*), PMED (*P*), and MIPROG (*IP*). The CD icon will appear with the software package designation where the problem analysis is assisted by one of these software programs. A database may be prepared for the problem if extensive data input is required. Where the problem can be solved without the aid of the computer (by hand), the hand icon is shown. If no icon appears, hand calculation is assumed.

1. Two plants are to serve three market points through one or two warehouses, as shown in Figure 13-16. Volume flowing either to or from each point, and the associated transportation rates, are given as follows.

Point No.	Point, *i*	Volume, V_i(cwt.)	Transportation Rate, R_i($/cwt./mi.)
1	P_1	5,000	0.04
2	P_2	7,000	0.04
3	M_1	3,500	0.095
4	M_2	3,000	0.095
5	M_3	5,500	0.095

a. Using the center-of-gravity method, find the approximate location for a *single* warehouse.
b. Using the *exact* center-of-gravity method, find the optimum *single* warehouse location.

Figure 13-16
Location of Plants and Markets with Grid Overlay



c. Evaluate the quality of the solutions obtained in terms of their optimality and usefulness. Evaluate the factors included or not included in the model. Explain how management might use the solution.
d. Find the optimum locations for *two* warehouses to serve these markets. Assume that each plant serves each warehouse in proportion to the market volume assigned to the warehouse.

2. The Care-A-Lot Hospital Group wishes to locate an outpatient clinic or clinics in a rural area of Africa. The construction costs and other considerations suggest that one or two centers would be about right. Since traveling is difficult for patients in this part of the world, proximity to such facilities often dictates their choice. Therefore, location is best determined based on weighted distance (number of patients times distance from the facility). Figure 13-17 shows the annual number of patients likely to visit the clinic(s) and their cluster locations. It is estimated that it costs an average of $0.75 per kilometer (prorated on the basis of a one-way trip) for a patient to travel or be transported to the clinic(s). This estimate is based on lost productivity, direct travel costs, and indirect travel expenses paid for by others.
   a. What is the best location for a single clinic?
   b. If two clinics are to be located, where is the best location for them?
   c. A clinic costs $500,000 (U.S. dollars) per year to equip and staff. This is paid through charitable contributions and government subsidies. On purely economic grounds, should the second clinic be built?
3. Bottoms-Up, Inc., is a small company that produces and distributes beer under the Old Wheez label. The company is examining the possibility of penetrating the North Shore City metropolitan area market. A plant location that would serve the area is sought. A grid overlay is placed over the selling area, as shown in Figure 13-18. North Shore City is area *E*. The suburbs surrounding *E* are designated as *A* to *I*. A market research study shows the following potential demand for Old Wheez.

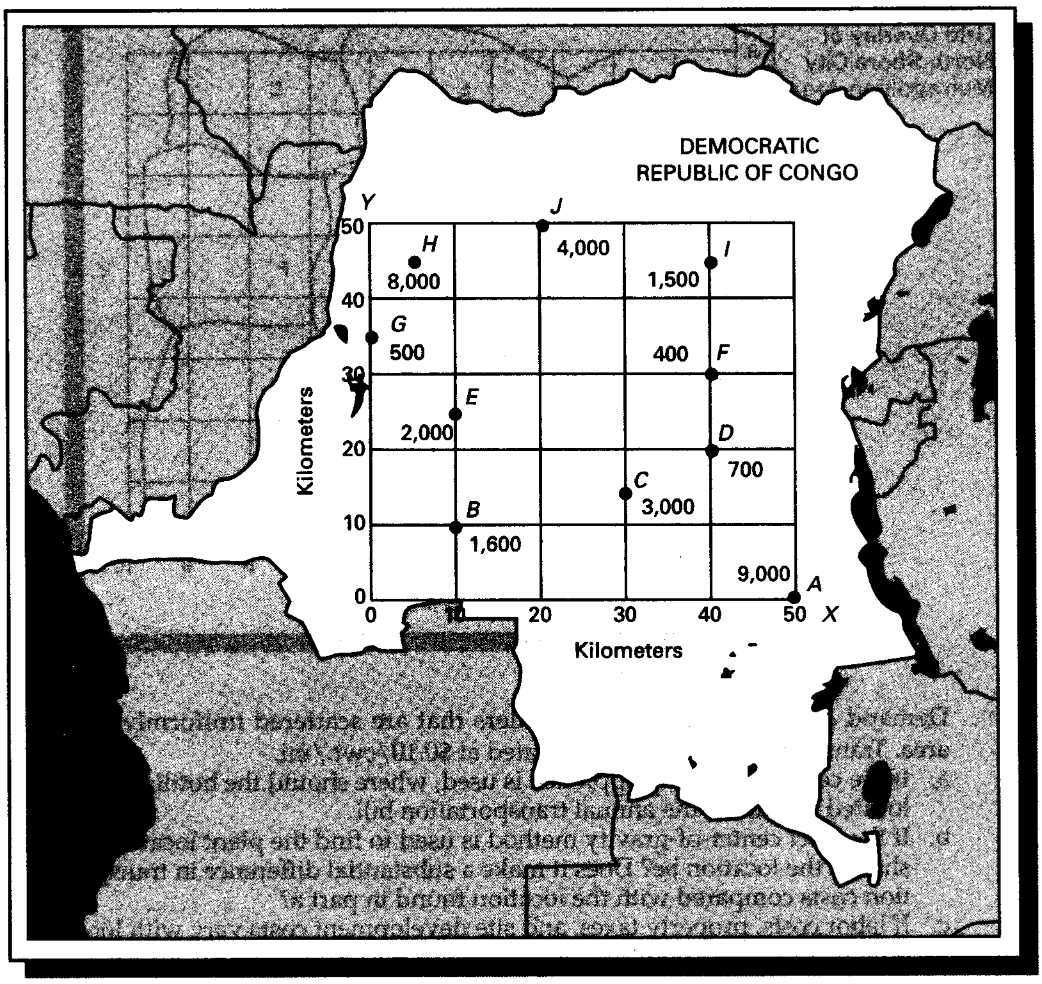
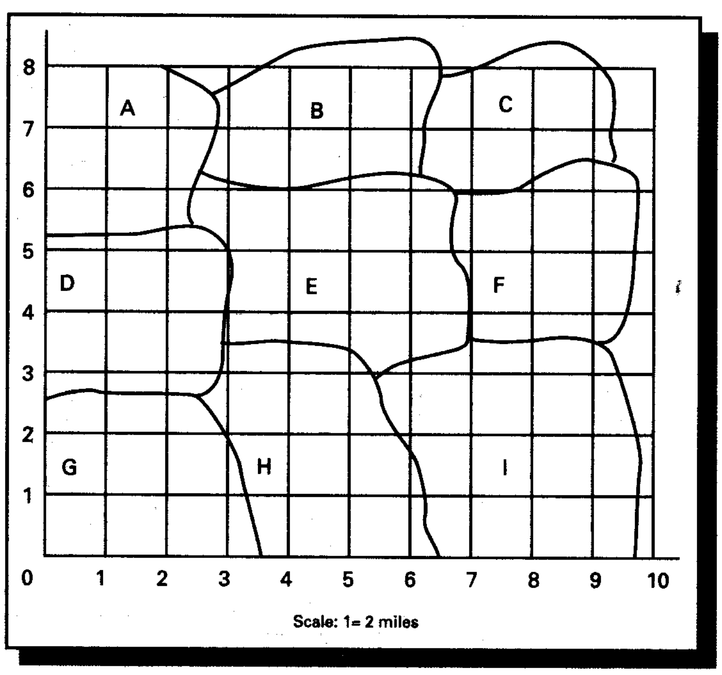


Figure 13-17 Grid Overlay on Patient Concentrations in a Region of Africa

Area	Annual Volume (cwt.)
A	10,000
B	5,000
C	70,000
D	30,000
E	40,000
F	12,000
G	90,000
H	7,000
I	10,000

Figure 13-18
Grid Overlay of North Shore City Metropolitan Area



Demand comes primarily from dealers that are scattered uniformly over the area. Transportation costs are estimated at \$0.10/cwt./mi.

a. If the center-of-gravity approach is used, where should the bottling plant be located? Estimate the annual transportation bill.

b. If the *exact* center-of-gravity method is used to find the plant location, where should the location be? Does it make a substantial difference in transportation costs compared with the location found in part a?

c. If labor costs, property taxes, and site development costs vary with location, how would you propose accounting for these additional costs when deciding the location?

4. Recall the problem presented in Figure 13-10. Resolve it, assuming that both warehouses are public warehouses and, therefore, no fixed costs apply. Summarize your answer in terms of customer, warehouse, and plant assignments.

5. Recall the problem presented in Figure 13-10. Resolve it, assuming that warehouse 2 can handle only 100,000 cwt. Warehouse 1 is expanded to handle an unlimited throughput. The plant capacities remain unchanged. How much of a cost penalty can be expected from this change?

6. Recall the problem presented in Figure 13-10. Resolve the problem, assuming that the fixed cost on warehouse 2 (W_2) is \$200,000 instead of \$400,000 per year.

7. Develop a list of factors that you think would be important in deciding the location of

a. A Goodwill collection center

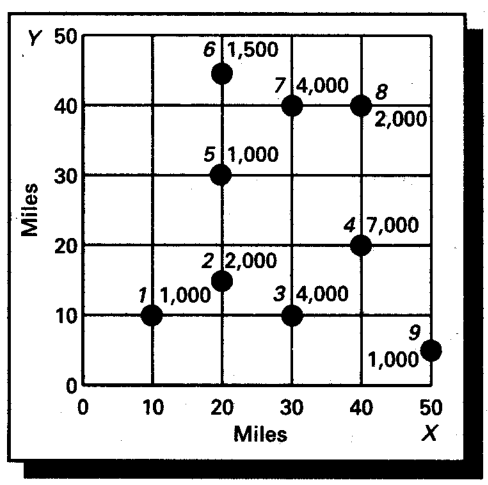
b. A Wendy's restaurant

c. An automobile assembly plant
d. A fire station

Also, indicate the weight that you would assign to each factor.

8. In the problem shown in Table 13-6 and Figure 13-13, suppose that the cost associated with moving from one location to another is \$300,000 instead of \$100,000. What location planning strategy would offer the maximum profits over the five-year planning horizon?
9. Recall the Environment Plus incinerator location problem and the data for the problem in the file PMED02.DAT of the PMED module in LOGWARE. Consider the following additional questions:
   a. How many facilities will give the lowest operating cost plus transportation cost? Where should these incinerators be located? Show that the best number of incinerators has been found.
   b. The company currently operates four incinerators at an annual operating and transportation cost of \$35 million. Chicago, Atlanta, Phoenix, and Denver are the sites. The cost to establish a new incinerator site is a one-time cost of \$6,000,000 per site. Is it economically reasonable for the company to establish the optimal number of sites found in part a?
   c. If the West Coast market of Los Angeles and Seattle were to grow by 10 times, would your answer to part a be different?
10. Suppose that Farmers' Bank wishes to serve the nine customer clusters as shown in Figure 13-19. It has proposed locating a branch (*A*) at coordinates $X_1 = 20$, $Y_1 = 20$. A competing branch (*B*) is located at coordinates $X_2 = 40$, $Y_2 = 30$. Farmers' Bank is to be a full-service branch with a relative size (attractiveness) index of 1. The competing branch is a partial-line facility (no ATM, no drive-through capabilities) with a size index of 0.7. The travel time for customers to a bank is approximated as $T(\text{hours}) = D/50$, where D = distance in miles. The average customer generates \$100 in gross annual revenue for a bank. The

Figure 13-19
Number of Potential Customers for Branch Banks in a Region



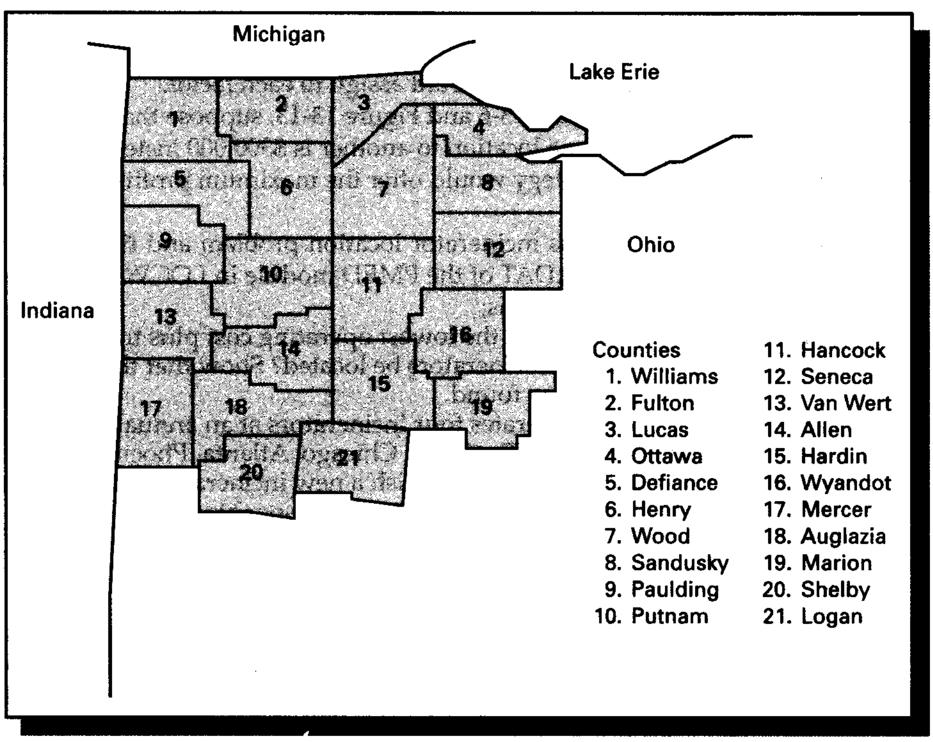


Figure 13-20 Northwest Ohio Counties for Ohio Trust Company Expansion

estimate of annual operating expense for Farmers' branch is $300,000, and the facility will cost $650,000 (20-year life) on land worth $100,000.

a. Apply Huff's retail gravity model to determine the branch's annual revenue. Assume $a = 2$.
b. Considering the level of investment required and the operating expenses, should the branch be constructed?
c. What additional information might you like to have before making a final decision?

11. The Ohio Trust Company wishes to expand its principal place of business locations to the northwestern counties of Ohio. The conditions for location were outlined in a previous example in Figure 13-15 on page 593. For the counties identified in Figure 13-20, find the minimum number of PPBs needed and the counties in which they should be located. (*Note:* A database is prepared for this problem in the module MIPROG.)

12. Biogenics is a start-up company that plans to produce biological materials used in medical research. Major customers for their products will be the large research hospitals located in major metropolitan areas. Customer location and projected annual sales are as follows.

No.	Customer	Latitude,°	Longitude,°	Sales, lb
1*	Boston	42.31	71.08	50,000
2*	New York	40.72	74.00	75,000
3	Washington	38.89	77.00	45,000
4*	Atlanta	33.75	84.38	65,000
5*	Miami	25.83	80.28	35,000
6*	Cleveland	41.48	81.66	25,000
7	Detroit	42.36	83.06	30,000
8*	Chicago	41.83	87.64	70,000
9	St. Louis	38.63	90.19	20,000
10*	Minneapolis	44.92	93.20	15,000
11	Kansas City	39.10	94.58	10,000
12*	Philadelphia	39.95	75.17	30,000
13*	Houston	29.78	95.38	25,000
14*	Dallas	32.98	96.78	20,000
15*	Phoenix	33.49	112.08	10,000
16*	Denver	39.73	104.98	15,000
17*	Seattle	47.63	122.33	10,000
18	Portland	45.46	122.67	10,000
19*	San Francisco	37.78	122.21	40,000
20*	Los Angeles	34.08	118.36	80,000

*Indicates a candidate site.

Products will be shipped UPS at a transport rate that averages \$0.05/lb/mile. It is estimated that the annual operating costs (*FOC*) for a laboratory (plant) are given from $FOC(\$) = (\$5,000,000\sqrt{N})/N$, where N is the number of laboratories being operated. The vendors for the materials used in the production process are assumed concentrated at Chicago. The purchase weight is the same as the sales weight. The transport rate from Chicago to the laboratories is estimated at \$0.02/lb/mile.

Determine the number and location of the laboratories to serve Biogenics' potential markets. Which customers should be served out of each site? Every customer location is a potential laboratory site, except St. Louis, Portland, Kansas City, Washington, Detroit, and Chicago vendors.

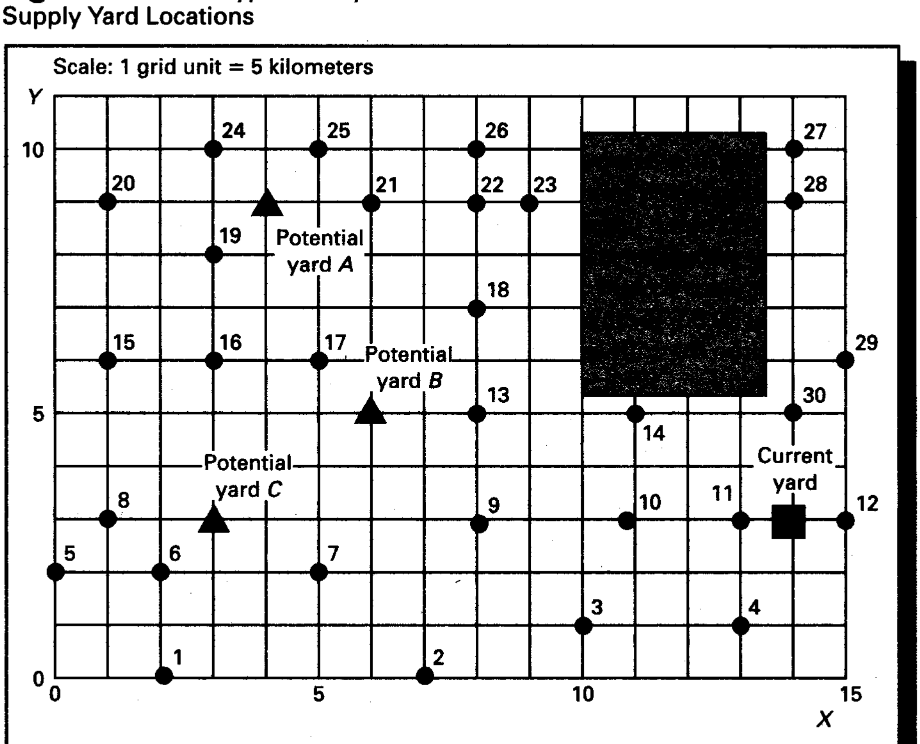
13. For the problem shown in Figure 13-5, suppose that the capacity restriction on warehouse 2 (W_2) for both products combined is 100,000 cwt. There is no capacity restriction on warehouse 1 (W_1). (*Note:* Data for the problem shown in Figure 13-5 are available in the MIPROG module of LOGWARE. Capacities are at the insertion point of Cap-W1/ZW1 and Cap-W2/ZW2 in the problem setup.)
14. Recalling the problem presented in Figure 13-5, how would the solution change if the following alterations were made to the problem setup?
    a. Demand for product 1 is doubled, but remains unchanged for product 2.
    b. The manufacturing cost for product 2 is raised to \$5/cwt. at plant 2 only.
    c. The handling cost for warehouse 2 is increased to \$4/cwt.

d. There is a limited capacity in plant 2 of 90,000 cwt. to produce product 1. Plant 1 capacity is increased from 60,000 cwt. to 150,000 cwt. There are no plant capacity changes for product 2.
e. Customer 2 for product 2 can no longer be served from warehouse 2.

Table 13-12 Times Between Zones for Globe Casualty Company, in Minutes

	To Zone									
From Zone	1	2	3	4	5	6	7	8	9	10
1	5	23	34	15	45	55	25	10	9	19
2		5	18	12	53	37	27	33	26	16
3			5	6	14	41	31	28	24	17
4				5	15	29	45	60	31	23
5					5	25	27	14	39	43
6						5	7	13	42	53
7							5	33	14	8
8								5	26	10
9									5	19
10										5

Figure 13-21 Typical Daily Demand Pattern with Current and Potential Supply Yard Locations



Resolve *separately* each of the above problem scenarios using the mixed integer linear programming approach. (*Note:* The problem setup for Figure 13-5 is available in the MIPROG module of LOGWARE.)

15. The Globe Casualty Company positions claims adjusters around a metropolitan area to respond quickly to insurance claims resulting from auto accidents, fires, crimes, and other such emergency situations. It is a competitive feature of the company's business for an adjuster to be on-site within 30 minutes of the time an accident is called in, so that customers feel they are being well served. The city has been divided into ten zones from which casualty calls originate and in which claims adjusters may be stationed. The response times in minutes between the ten zones are shown in Table 13-12. To meet the 30-minute response time, how many claims adjuster stations should be established, and in which zones should they be located?

16. A building supply firm supplies materials to construction sites throughout the metropolitan area of Mexico City, Mexico. Daily delivery trucks are dispatched from a materials yard. A typical daily demand pattern is shown in Figure 13-21, where a grid is overlaid on the metropolitan area. The map-scaling factor is 1 coordinate unit = 5 kilometers with a circuity factor of 1.44 to convert straight-line distance to approximate road distance. Demand is given in kilograms of merchandise in Table 13-13.

Table 13-13 Customer Demand Volume and Coordinate Locations with Yard Coordinates

Stop	X	Y	Volume, kg	Stop	X	Y	Volume, kg
1	2	0	300	16	3	6	300
2	7	0	250	17	5	6	150
3	10	1	600	18	8	7	275
4	13	1	175	19	3	8	375
5	0	2	100	20	1	9	475
6	2	2	375	21	7	9	150
7	5	2	400	22	8	9	475
8	1	3	50	23	9	9	325
9	8	3	100	24	3	10	350
10	11	3	200	25	5	10	225
11	13	3	350	26	8	10	250
12	15	3	100	27	14	10	300
13	8	5	200	28	14	9	200
14	11	5	450	29	15	6	150
15	1	6	225	30	14	5	50
Current Yard	14	3					
Yard *A*	4	9					
Yard *B*	6	5					
Yard C	3	3					

Trucks operate with a variable cost of 2.5 pesos per kilometer, driver's wages are 90 pesos per day, and a truck is amortized at 200 pesos per day. The materials are stored in open yards and in buildings at locations shown in Figure 13-21. The current materials yard from which trucks are dispatched operates at a cost of 350 pesos per day. The company is considering moving its operations. The potential materials yards have an estimated operating expense, including the moving expense from the current location, of 480 pesos per day for *A*, 450 pesos per day for *B*, and 420 pesos per day for *C*. There are ten trucks available, each with a hauling capacity of 1,000 kg, but not all may be needed for meeting average demand. Trucks travel over their routes at an average speed of 32 kph. After noon, drivers are permitted a one-hour lunch break and they usually return to the yard before needing additional breaks. Based on a company policy, routes should be no longer than ten hours in a day, and trucks are not to leave the yard before 8 A.M. If a truck has a short route and returns for reloading and rerouting, 1.5 hours for loading is required. Customers have a time window for delivery of between 8 A.M. and 5 P.M. Time for unloading at a customer location is estimated as 15 minutes plus 0.1 times the stop volume in kilograms.

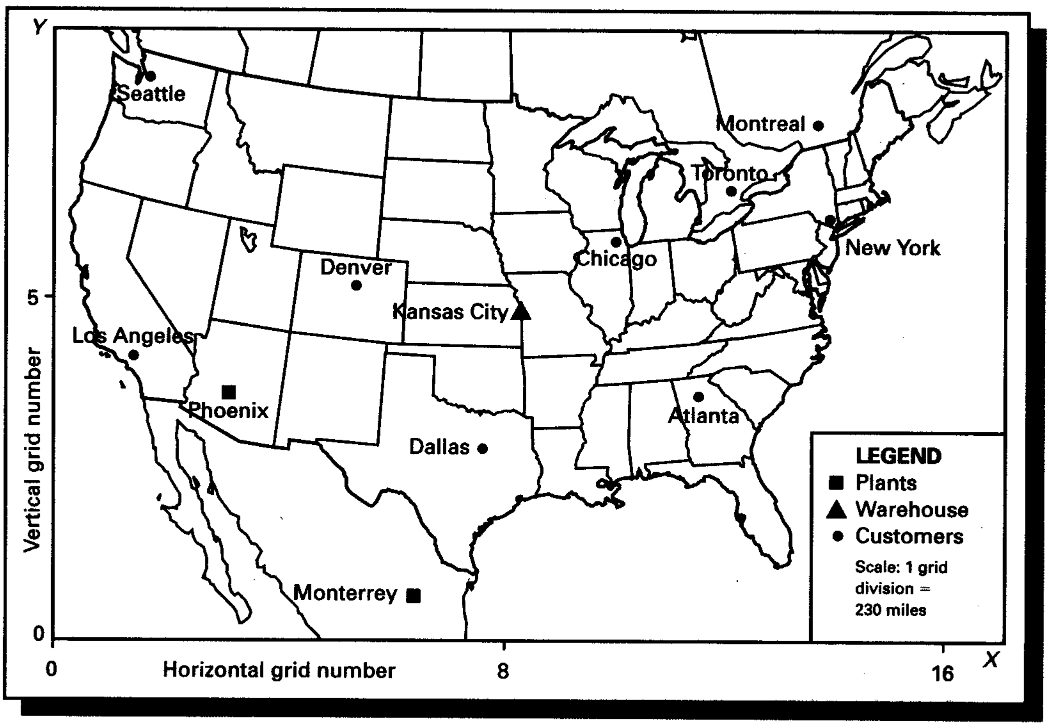
Which yard location is most economically attractive?

Superior Medical Equipment Company

Superior Medical Equipment Company supplies electrical equipment that is used as components in the assembly of MRI, CAT scanners, PET scanners, and other medical diagnostic equipment. Superior has production facilities in Phoenix, Arizona, and Monterrey, Mexico. Customers for the components are located in selected locations throughout the United States and Canada. Currently, a warehouse, that receives all components from the plants and redistributes them to customers, is located at Kansas City, Kansas. Figure 1 shows the geographical placement of these facilities.

Superior's management is concerned about the location of its warehouse since its sales have declined due to increasing competition and shifting sales levels among its customers. The lease is about to expire on the current warehouse, and management wishes to examine whether it should be renewed or warehouse space at some other location should be leased. The warehouse owner has offered to renew the lease at an attractive rate of $2.75 per sq. ft. per year for the 200,000 sq. ft. facility. It is estimated that any other location would cost $3.25 per sq. ft. for a similar-size warehouse. A new or renewed lease

Figure 1 Location of Superior's Plants, Warehouse, and Customers



Plant Location	Annual Volume, cwt.[b]	Transport Rate, $/cwt.	Distance, Miles	Grid Coordinates[a] X	Y
Phoenix	61,500	16.73	1163	3.60	3.90
Monterrey	120,600	9.40	1188	6.90	1.00
Total	182,100				

[a]Miles = 230 × coordinate distance
[b]Cwt. = 100 lb.

Table 1 Volume, Rate, Distance, and Coordinate Data for Shipping from Plants to the Kansas City Warehouse in Truckload Quantities (Class 100) for the Most Recent Year

will be for five years. Moving the inventory, moving expenses for key personnel, and other location expenses would result in a one-time charge of $300,000. Warehouse operating costs are expected to be similar at any location.

In the most recent year, Superior was able to achieve sales of nearly $70 million. Transportation costs from the plants to the Kansas warehouse were $2,162,535, and from the warehouse to customers were $4,819,569. One million dollars was paid annually as warehouse lease expenses.

To study the warehouse location question, the data shown in Tables 1 and 2 were collected.

Although transport costs are not usually expressed on a $/cwt./mile basis, given that the outbound transportation costs for the most recent year were $4,819,569, the weighted average distance of the shipments was 1128 miles, and the annual volume shipped was 182,100 cwt., the estimated average outbound rate from a warehouse would be $0.0235/cwt./mile. ■

Table 2 Volume, Rate, Distance, and Coordinate Data for Shipping from the Kansas City Warehouse to Customers by Truck in 5,000 lb Quantities (Class 100) for the Most Recent Year

Customer Location	Annual Volume, cwt.	Transport Rate, $/cwt.	Distance, Miles	Grid Coordinates X	Y
Seattle	17,000	33.69	1858	0.90	9.10
Los Angeles	32,000	30.43	1496	1.95	4.20
Denver	12,500	25.75	598	5.60	6.10
Dallas	9,500	18.32	560	7.80	3.60
Chicago	29,500	25.24	504	10.20	6.90
Atlanta	21,000	19.66	855	11.30	3.95
New York	41,300	26.52	1340	14.00	6.55
Toronto	8,600	26.17	1115	12.70	7.80
Montreal	10,700	27.98	1495	14.30	8.25
Total	182,100				
Kansas City				8.20	6.00

QUESTIONS

1. Based on information for the current year, is Kansas City the best location for a warehouse? If not, what are the coordinates for a better location? What cost improvement can be expected from the new location?
2. In five years, management expects the Seattle, Los Angeles, and Denver markets to grow by 5 percent, but the remaining markets to decline by 10 percent. Transport costs are expected to be unchanged. Phoenix output will increase by 5 percent, and Monterrey's output will decrease by 10 percent. Under these new conditions, would your decision about the warehouse location change? If so, how?
3. If by year 5 increases are expected of 25 percent in warehouse outbound transport rates and 15 percent in warehouse inbound rates, would your decision change about the warehouse location?
4. If the center-of-gravity method is used to analyze the data, what are its benefits and limitations for locating a warehouse?

Ohio Auto and Driver's License Bureaus

As a member of the planning commission for the state of Ohio, Dan Rogers was concerned about how the state could conserve tax dollars in providing services to its residents. Projected cutbacks in federal funds to the state, difficulties in increasing tax rates, and the general upward trend in operating costs encouraged a careful examination of how costs might be reduced throughout the state. Dan had a particular interest in how auto and driver's licensing bureaus might be operated more efficiently.

Dan thought that a study should be conducted to examine the locations, sizes, and number of license bureaus statewide. License bureaus issue motor vehicle license tags, driver's licenses, and motor vehicle registrations. The bureaus are located throughout the state for the convenience of the residents; however, the number of bureaus must be limited, due to the fixed costs associated with opening and maintaining a bureau location and the cost of operating it. Since population has shifted from central cities to the suburbs throughout the state and the network of license bureaus has not been evaluated in a number of years, Dan believed that there were too many bureau locations and they were likely in the wrong locations. He thought that not only could costs be reduced, but service to the residents improved.

The Cleveland, Ohio, metropolitan area represents a typical service area that Dan believed would be a good test region to see if improvements might be made. A map of the region is shown in Figure 1. A linear grid was overlaid on the map, with grid divisions approximately 2.5-miles square. Population was used to represent the relative activity on a license bureau site. Approximate population levels for each grid block are given in Table 1, with clustering at the grid center. Locations of the existing bureaus are shown on the map.

Residents usually select a bureau location that is closest to their residence. Except for motor vehicle tags, which may be acquired through a mail-in program, there is no competition for a bureau's services. A major element of customer service is how far a resident must travel to a bureau.

Dan made some rough estimates of the costs involved. Operating costs for a site included rental charges on the space, salaries of personnel, and utilities. Space, staffing, and

Figure 1 Cleveland, Ohio, Area Auto and Driver's License Bureau Locations